Macroeconomics (ECO 558)
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You may answer in English or French. No documents allowed (except for a paper dictionary). This exam contains three sections: A, B and C. Each section is worth 7 points.

Section A (7 points)

The exercises of section A are graded on 9 points. However, you cannot be awarded more than 7 points in total for section A.

A1: The dynamics of public debt (3 points)

Short answers please.

1. Recall that the dynamics of a government debt is

$$B_t = (1 + R_{t-1})B_{t-1} + G - T_t - (M_t - M_{t-1})$$

with $B_t, R_{t-1}, G - T_t$ and M_t, being the respectively the nominal debt, the nominal interest rate, the primary deficit and the stock of base money.

Divide the debt by nominal GDP P_tY_t and show the role of real growth $g_t = \frac{Y_t}{Y_{t-1}} - 1$, inflation $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and the nominal interest rate R_{t-1} in the dynamic of the debt.

2. Recall the quantity theory of money $M_tv = P_tY_t$,with neutrality of Money with repect to and the Fisher theory of the long term interest rate $R_{t,t+k} = \rho + E(\pi_{t+k})$, discuss what the effects of printing euros to buy Italian or French public debt could be.

   - Why would such a policy risk alleviating the pressure on governments to limit deficits?
   - Why does is the ECB reluctant to finance deficits with printing money?

3. Consider now that the tax income are uncertain and its cumulative distribution is given by $F(T_t)$.We assume that if $T_t < (1 + R_{t-1})B_{t-1} + H$ with $H = -B_t + G - (M_t - M_{t-1})$, the government defaults.

Assuming that investors are risk neutral, they want that the return on lending to this government compensate for the default risk: $p \times 0 + (1-p) \times R_{t-1} = \bar{R}$,where p is the probability that the governement will default.

- Show how the probability of default depends on linearly on R_{t-1}.
- We have also that the government will default when $T_t < (1 + R_{t-1})B_{t-1} + H$. Hence $p = F((1 + R_{t-1})B_{t-1} + H)$. Assuming that the density distribution of T_t has a bell shape, $F(T_t)$ has a S shape. Draw the intersection(s) of $F(T_t)$ and arbitrage relation that ties p to R_{t-1}.
- How many equilibrium can there be? Does this lead to to reconsider the benefits of the central bank interventions to limit the level of interest rates on the Italian debt?

A2: A simple New-Keynesian model (6 points)

Assume a two period model where household supply labor L and consume C at each period. Their utility is as follows

$$U_H = \frac{C_1^{1-1/\sigma}}{1-1/\sigma} - \frac{L_1^{1+\eta}}{1+\eta} + \beta[\frac{C_2^{1-1/\sigma}}{1-1/\sigma} - \frac{L_2^{1+\eta}}{1+\eta}]$$

under the budget constraint

$$P_1C_1 + \frac{P_2C_2}{1+i} = W_1L_1 + \frac{W_2L_2}{1+i}$$

W, β, i stand for wages the discount rate and the nominal interest rate.

1. Using a lagrangian approach, derive the optimal intratemporal conditions that relate consumption and labor in each period to the real wage of that period.
2. Derive the intertemporal optimal allocation of consumption and show of the response of savings from period 1 to period 2 depends on the intertemporal elasticity of substitution σ. To proceed, you may recourse to defining C_2 from the budget constraint and substitute it into the utility function, and then derive the optimal level of C_1. Represent an agregate demand curve that relates $p_1 = \log(P_1)$ to $c_1 = \log(C_1)$, in a plan (c_1, p_1) which is consistent with this optimal intertemporal allocation of consumption.
3. Assume that producers use a linear technology in labor to produce the consumption good in each period: $C_i = AL_i$, and that if they can reset prices, they would apply a constant mark up to marginal cost: $P_1^R = \frac{W_1}{A}(1+\mu)$. Use conditions obtained in

question 1 to define the natural level of produced consumption in period 1 $C_{1,n}$ as the level where all producers can reset prices in period 1.

4. Now considering that only a proportion α of producers can reset prices, while $1-\alpha$ cannot and set prices to P^e, use again the condition obtained in question 1 to derive that p_1^R the log of prices that can be reset depends on $(c_1 - c_{1,n})$.

5. Using that the log of aggregate prices $p_1 = \alpha p_1^R + (1-\alpha)p^e$, derive the aggregate Philips curve between p_1 and $(c_1 - c_{1,n})$. Plot this aggregate Philips curve in the plan (c_1, p_1).

6. Represent the AD curve and the Philips curve of period 1 for

σ	η	α
2	2	1/3

Assume that, starting from the equilibrium of the AD curve and the Philips curve, the central bank lowers the interest rate by 1%. Describe the qualitative effects on c_1 and p_1.

- How would these effects change qualitatively if $\eta = 4$? Give a economic intuition of the result.
- How would these effects change qualitatively if $\alpha = 9/10$?Give a economic intuition of the result.

Section B: A Ramsey Model with Heterogeneous Households (7 points)

Consider an economy populated by J equal-sized infinitely lived household. Each household is indexed by $j \in \{1, 2, ..., J\}$. Household j's objective is to maximize its lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t(j) + \gamma(j)g_t)^{1-\theta} - 1}{1-\theta},$$

where β is the discount factor and θ is a preference parameter, which are both common to all households, $c_t(j)$ is household j's consumption at time t, g_t is the amount of publicly provided goods at time t and $\gamma(j) \in [0, +\infty)$ is household j's preference for the publicly provided goods. Households are also heterogeneous in productivity and in wealth. Let $a_t(j)$ denote household j's holding of risk-free assets at time t and let $\pi(j)$

denote its productivity. For simplicity, workers' average productivity is normalized to 1, i.e. $\sum_{j=1}^{J} \pi(j)/J = 1$. Household j's flow budget constraint is given by:

$$a_{t+1}(j) = (1 + r_t)a_t(j) + w_t\pi(j) - c_t(j),$$

where r_t is the exogenous net risk-free interest rate at t and w_t is the exogenous wage rate per efficiency unit of labor at t. Finally, the initial wealth of household j, $a_0(j)$, is exogenously given.

1/ Derive household j's no-Ponzi condition which guarantees that its intertemporal budget constraint is satisfied. (1 point)

2/ Derive the consumption Euler equation for household j. (1.5 points) [Hint: You can consider without proof that, for all t, $a_t(j)$ does belong to a compact set.]

3/ Let c_t and A_t denote aggregate consumption and wealth at time t, respectively. Thus, $c_t = \sum_{j=1}^{J} c_t(j)$ and $A_t = \sum_{j=1}^{J} a_t(j)$. Derive the equations that implicitly characterize the behavior of aggregate consumption and of aggregate wealth (for exogenously given time paths of the wage rate, of the interest rate and of publicly provided goods). What is the effect of the heterogeneity of households on the aggregate behavior of the economy? (1.5 points)

4/ For the rest of this exercise, assume that the quantity of publicly provided goods is equal to zero, i.e. $g_t = 0$ for all t. Show that, at any time t, household j chooses a consumption level equal to:

$$c_t(j) = \mu(t) \left[\tilde{w}_t\pi(j) + (1 + r_t)a_t(j)\right],$$

where $\tilde{w}_t$ denotes the present value of wages at t, i.e. $\tilde{w}_t = w_t + \frac{w_{t+1}}{1+r_{t+1}} + \frac{w_{t+2}}{(1+r_{t+1})(1+r_{t+2})} + ...$, and $\mu(t)$ is a time varying coefficient that you need to determine. (1 point)

5/ Show that:

$$\frac{a_{t+1}(j)}{A_{t+1}/J} - \frac{a_t(j)}{A_t/J} = \frac{w_t - \mu(t)\tilde{w}(t)}{A_{t+1}/J} \left(\pi(j) - \frac{a_{t(j)}}{A_t/J}\right).$$

Interpret this expression. (1 point)

6/ Assume that, instead of remaining constant throughout, the productivity of households fluctuates randomly over time, without affecting the economy-wide average level of productivity at any time t (which therefore remains equal to 1). Furthermore, assume that the risk-free asset remains the only financial asset in the economy. In that context, describe intuitively how household heterogeneity affects the aggregate behavior of the economy. (1 point)

Section C: The Term Structure of Interest Rates (7 points)

Consider a pure exchange economy with identical consumers. In each period t, three assets are available: a risky asset and two risk-free bonds with different maturities. Thus, at the beginning of time t, the representative consumer holds a quantity S_t of risky asset which yields an exogenous stream $\{D_s\}_{s=t}^{\infty}$ of Markovian dividends. He also holds a quantity B_t^1 of one-period bond paying one unit of consumption good at time t and a quantity B_t^2 of two-period bond paying one unit of consumption good at $t+1$.[1] The price of the risky asset is P_t at t. The risk-free gross return is R_t^1 between t and $t+1$ and R_t^2 between t and $t+2$. The representative consumer's problem at time 0 is therefore given by:

$$\max_{\left\{C_t, S_{t+1}, B_{t+1}^1, B_{t+2}^2\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t)\right]$$

$$\text{subject to} \quad C_t + P_t S_{t+1} + \frac{B_{t+1}^1}{R_t^1} + \frac{B_{t+1}^2}{R_t^2} \leq (D_t + P_t) S_t + B_t^1 + \frac{B_t^2}{R_t^1}$$

$$S_0 = 1,\ B_0^1 = 0 \text{ and } B_0^2 = 0$$

where C_t is the consumer's consumption at t, β is his discount factor and $u(.)$ is his instantaneous utility from consumption.

Finally, the net supply of risky asset at any time t is equal to 1 and the net supply of risk-free bond of each maturity is equal to 0. At any time t, the aggregate supply of goods is equal to the level D_t of dividends.

1/ Explain the representative consumer's flow budget constraint. (1 point)

2/ Derive the equilibrium price P_t of the risky asset. (1.5 points) [Hint: You can rule out bubbles without proof.]

3/ Compute explicitly the gross risk-free return R_t^1 on one-period risk-free bonds and the return R_t^2 on two-period bonds. (1 point)

[1] Thus, both types of bonds held at the beginning of time t were bought at $t-1$.

4/ Long term gross returns are sometimes thought to be equal to the product of short term returns, which would imply:

$$\frac{1}{R_t^2} = \frac{1}{R_t^1}\mathbb{E}_t\left[\frac{1}{R_{t+1}^1}\right].$$

Under what conditions is this true? Derive an expression relating these two terms and explain why, in general, they are not equal to each other? Which of the two terms is higher when dividends are independently and identically distributed over time? Provide an intuition. (2 points)

5/ Define the yield as the per-period return on an asset. Thus, the gross yield at t is R_t^1 on a one-period bond and $\sqrt{R_t^2}$ on a two-period bond. Assuming that dividends are independently and identically distributed over time, under what condition is the yield curve upward sloping, i.e. when is $\sqrt{R_t^2} > R_t^1$? Provide an interpretation for your result. (1.5 points)