Interpretating Macroeconomic Time Series

Sciences-Po M2 2009-2010

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6 Monday lectures from 17:00 to 19:00
2/11 in J210 (13 rue de l'Université)
9/11 in C901 (9 rue de la Chaise)
16/11 in J210 (13 rue de l'Université)
23/11 in A31 (27 rue St Guillaume)
7/12 in J210 19:15 (A confirmer)
14/12 in K720 (117, Bvd St Germain)

6 Thursday TDs from 19:15 to 21:15, by Aurélien Poissonier (INSEE-CREST)

Class material to be posted weekly at www.benoitmojon.com (teaching)

List of lectures

Lecture 1 and 2: Overview and Filtering
Lecture 2 and 3: RBC calibration
Lecture 3 and 4: Toward the standard DSGE model
Lecture 4 and 5 : VAR estimation
Lecture 5 : VAR identification
Lecture 6 : Maximum likelihood estimation of a DSGE

General remarks

1. I take a "user" point of view. I describe why and how we use economic models and econometric tools.

2. Repetition is the essence of pedagogy: they will be overlaps with other lectures you followed already or will follow. This lecture sequence is one take on this field of macro.

3. Hardly any final truth in economics: most of the elements reviewed in the class are continuously questioned by ongoing research. The most frustrating you find these concepts and tools the more likely you will be motivated to elaborate newer, better ones.

1 Lecture 1 to 3: Filtering and RBC calibration

Outline

1. **Introduction**
2. **Filtering and definition the facts that the model should replicat**
3. **How do we simulate data from a calibrated model?**

Reading list

Baxter and King (RES-1999)
King and Rebello (Handbook-1999, chap 14)

Stock and Watson (Handbook-1999, chap 1)
Agresti and Mojon (2003)

Exercise for November 9

Replicate Agresti and Mojon Table 1 for two samples, either US or EA data
1984-2006
1984-2009
Does the sub-prime crisis change the picture?
(data and programs are avaible on the net: FRED- St Louis Fed; Eurostat; google for Matlab programs; see also www.benoitmojon.com/ teaching)

1.1 Introduction

The general purpose of this sequence of lectures is to review widely used approaches to analyse macroeconomic time series.

$$Y_t = f(Y_t, Y_{t-1}, ..., X_t, X_{t-1}, ..., \Theta, \varepsilon_t)$$

- Y_t is a vector of observable economic variables, say GDP, the GDP deflator and the short term interest rate taken at successive quarterly realisations. We consider these variables endogenous.
  - X_t is a vector of exogenous variables
  - Θ is a vector of fixed parameters
  - ε_t are residuals, innovations or interpretable shocks.

Calibration is the first method we review to analyse the data. It consists of chosing Θ on the basis of knowledge not directly based on the variance (information) of the time series.

Example 0:

$$\begin{pmatrix} dy_t \\ dp_t \\ r_t \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.5 & 0.5 & 0 \end{bmatrix} \begin{pmatrix} dy_t \\ dp_t \\ r_t \end{pmatrix} + \begin{bmatrix} 0.5 & c & -c \\ b & 0.5 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{pmatrix} dy_{t-1} \\ dp_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

Here we set the vector of model's parameters Θ arbitrarly

We assume:

- a sort of Taylor rule with interest rate smoothing of 0.8 and the canonical Taylor coefficients
- A backward looking IS curve with real interest rate elasticity of $-c$ and inertia of 0.5
- A backward looking Philips curve with slope b and inertia of 0.5

Any view on the value of b and c?

At this stage, let's calibrate $b = 0.2$ and $c = 0.3$ like we did for the other elements of θ , i.e. completely by chance.

An alternative approach to this ad hoc calibration could be estimation.

What would estimation provide us with? Compare the properties of the vector of ε_t when

- estimating the above system with OLS
- chosing the parameters by calibration.

1.2 How do we define the facts that the model should replicate (filtering)?

Main reference: Baxter and King (1999)

Other references: Stock and Watson HB-1999; Agresti-Mojon, 2003

1.2.1 Objectives of filtering

What economic fluctuations reflect the business cycle? What variations owe to phenomena that pertain to temporary imbalance between demand and supply? What can be influenced by macroeconomic stabilisation policies?

- Basic intuition 1: long term growth is a slow moving (low frequency) that owes to technical progress and its diffusion.
- Basic intuition 2: high frequency variations may be dominated by not so interesting stuff: measurement error, seasonality.
- Basic intuition 3: trending variables are hard to observe, analyse

We can therefore define objectives of filtering:

1. detrend if necessary
2. focus on fluctuations that correspond to cycles of length comprised between 6 and 32 to 40 quarters
3. avoid phase shift
4. easy to implement
5. *time invariance of the filter*
6. *approximate the true filter that we are after*

Well known filters

- first, fourth, twelfe difference:

$$\begin{aligned} dy_t &= y_t - y_{t-1} \\ d^4 y_t &= y_t - y_{t-4} \\ d^{12} y_t &= y_t - y_{t-12} \end{aligned}$$

advantages: detrend, intuitive, widely used

issues: keep high frequency noise, shift the phase

- The Hodrick Prescott filter

$$y_t^c = y_t - y_t^{trend} = y_t - \sum_{j=-K}^{K} a_j y_{t-j}$$

$$\min \sum \left\{ \left(y_t - y_t^{trend}\right)^2 + \lambda \left(y_t^{trend} - y_{t-1}^{trend}\right)^2 \right\}$$

advantages: detrend, most widely used method

issues: keep high frequency noise, arbitrary λ, not so good at begining and end of time series

- The BK band pass filter (my favourite!)

$$y_t^c = y_t - y_t^{trend} = y_t - \sum_{j=-K}^{K} a_j y_{t-j}$$

a_j are based on a frequency domain representation: keep only the frequencies of interest.

advantage: intuitive, eliminates both trend and high freq noise

issues: truncation implies loss of observations

1.2.2 Intuition from the frequency domain

Any time series can be represented in the frequency domain, i.e. as sums of cycles of different frequencies. The highest frequency is a cycle of 2 periods. The lowest frequency is a cycle of infinite length. In between, cycles of 4, 5,...., 100 periods.

Likewise, we can decompose the variance of any time series in the frequency domain using spectral densities.

Example: Analyse Chart 1 of Agresti-Mojon (Spectral densities of GDP growth for the euro area and the US)

Let's consider the representation of a mean zero stationary time series in the frequency domain:

$$y_t = \int_{-\pi}^{\pi} \xi(\omega) d\omega$$

A linear filter of y_t can be represented in the time and the frequency domains as

$$y_t^* = \sum_{j=-K}^{K} a_j y_{t-j}$$

$$y_t^* = \int_{-\pi}^{\pi} \alpha(\omega)\xi(\omega)d\omega$$

with $\alpha(\omega) = \sum_{j=-K}^{K} a_j e^{-i\omega j}$, that is, the weights of each frequency ω in the filter.

Because the frequencies are orthogonal to one another, we can represent the variance of the filtered series in the frequency domain:

$$Var(y_t^*) = \int_{-\pi}^{\pi} |\alpha(\omega)|^2 \, Var(\xi(\omega))d\omega$$

Basic intuition: if we are only interested in some frequencies, we will set the weights of the frequencies of interest to 1 and the weights of the frequencies we want to get rid of to 0:

$$\beta(\omega) = 1 \text{ for } \omega_{\min} < \omega < \omega_{\max}$$
$$\beta(\omega) = 0 \text{ for } \omega_{\min} > \omega \text{ and } \omega > \omega_{\max}$$

Implementation: define the weights of the linear filter by applying the inverse Fourier transform of the frequency domain representation:

$$b_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} \beta(\omega)e^{-i\omega j}d\omega$$
$$b_0 = \frac{\omega_{\min}}{\pi}$$
$$b_j = \frac{\sin(j \times \omega_{\min})}{j \times \pi}, \text{ for } j = 1, 2, ...N$$

The ideal filter would be an infinite moving average, which is not pratical, if implementable at all. In practise, we will approximate by truncation at$+/-K$ lags. We want to minimise the distance between the ideal infinite order filter and the truncated one. If we give equal weights across frequencies:

$$Min\frac{1}{2\pi}\int_{-\pi}^{\pi} |\alpha_K(\omega) - \beta(\omega)|^2 \, d\omega$$

A very convenient property of these filters, is that the ideal approximation of the infinite length filter simply consists of setting

$$a_j = b_j \text{ for } j < K$$
$$a_j = 0 \text{ for } j > K$$

Representation of the effects of truncation: see BK figure 2 and comment.

Compare the BK(8,32,12) and the Hodrick-Prescott filter: see Figure 1 in KR-HB.

Other properties:

- symmetry of the moving average of the BK prevents phase shifts.
- Sum of weights = 0 assures detrending.

1.2.3 Is the euro area an "economy"? What does its business cycle look like?

Answering this question is the purpose of Agresti and Mojon (2003).
Brief overview of the article.

1.3 How do we simulate data from a calibrated model?

(based on King and Rebelo, HB, section 3 and 4, KR-HB thereafter)

We now will follow in particular the calibration used to simulate the plain vanilla RBC model. This for 2 reasons:

1. It is the backbone of the DSGE models that are widely and increasingly used in central banks to analysie monetary policy and the business cycle.
2. It is the simplest way to illustrate the distance between a theoretical model and the data.

The claim of Prescott and the RBC school is that real shocks (productivity/ technology) accounts for the bulk of business cycle fluctuations. They have showed that a simple NeoClassical model with stockastic productivity shocks can generate fluctuations that are very similar to the ones observed in the data.
[side on Daniele, Marcus and the Friedman tree]

How do we simulate data from a calibrated model?

How do we define the facts that the model should replicate (long aside on filtering)?

1.3.1 The behavioral equations of the model

$$\sum \beta^t u(c_t, L_t)$$

$$\begin{aligned} N_t &= 1 - L_t \\ y_t &= A_t F(k_t, N_t) \\ y_t &= c_t + i_t \\ \gamma k_{t+1} &= i_t + (1-\delta) k_t \end{aligned}$$

where lower case letters are scaled by ($X_t = \gamma X_{t-1}$) the labor augmenting, deterministic productivity (see KR-HB)

we can combine the constraint in order to focus on two dual/shadow prices associated respectively to the intratemporal substitution of labor and leisure (the real wage) and the intertemporal substition (the "discount factor") between consumption and capital accumulation.

The Lagrangian associated to this intertemporal optimisation of utility can be written as follows:

$$\begin{aligned} L &= \sum \beta^t u(c_t, L_t) \\ &+ \sum \beta^t \lambda_t \left[A_t F(k_t, N_t) + (1-\delta) k_t - c_t - \gamma k_{t+1} \right] \\ &+ \sum \beta^t \omega_t \left[1 - L_t - N_t \right] \end{aligned}$$

The associated FOC are
consumption: $u'_c(c_t, L_t) = \lambda_t$
leisure: $u'_L(c_t, L_t) = \omega_t$
labor: $\lambda_t A_t F'_N(k_t, N_t) = \omega_t$
capital accumulation:
$\beta \lambda_{t+1} \left[A_t F'_k(k_{t+1}, N_{t+1}) + (1-\delta) \right] = \gamma \lambda_t$

Interpretation.

1.3.2 Calibration of the (quarterly) model (KR-HB, section 4)

At the steady state

- Discount factor $\beta = 0.99$
- Assume CobbDouglas production function. This implies that factor shares of income are equal to their proportion in the production function:
  $\frac{wL}{PY} = \alpha$ with $y = Ak^{1-\alpha} N^{\alpha}$
  Using US national accounts: $\alpha = 0.66$

- The deterministic rate of quarterly productivity progress set fromY/N in long period, $\gamma = 1.004$
- Quarterly depreciation: $\delta = 0.025$
- This implies a capital stock that must satisfy

$$\frac{k}{N} = \left[\frac{(1-\alpha)A}{r+\delta}\right]^{1/\alpha} = \left[\frac{(1-\alpha)A}{\frac{\gamma}{\beta}-1+\delta}\right]^{1/\alpha}$$

- Assume a functional form for the utility function

$$u(c_t, L_t) = \log(c_t) + \frac{\theta}{1-\eta}(L_t^{1-\eta} - 1)$$

Least satisfactory part (in my view) of the RBC calibration exercise relates to the time worked: 40 hours a week is $40/(7 \times 24) = 0.238$!!!!!!!!; (why not $8/24 = 0.33$?)

(Footnote 31 in KR-HB, why can't this be satisfactory)

$$\theta = \alpha \frac{y}{c}\frac{L^\eta}{N} = 0.66 \times \frac{1}{0.6}\frac{(1-0.238)^1}{0.238} = 3.52$$

sensitivity example 1: vary the assumed working time

$$\theta^{other} = \alpha \frac{y}{c}\frac{(1-N^{other})^\eta}{N^{other}} = 0.66 \times \frac{1}{0.6}\frac{(1-0.33)^1}{0.33} = 2.23$$

sensitivity example 2: include government consumption in "consumption"

$$\theta^{yet\ another} = \alpha \left[\frac{y}{c}\right]^{other}\frac{L^\eta}{N} = 0.66 \times \frac{1}{0.75}\frac{(1-0.238)^1}{0.238} = 2.88$$

- Computation of the Steady State

$$\begin{aligned}
N^{SS} &= 0.2 \\
L^{SS} &= 1 - N^{SS} = 0.8 \\
A^{SS} &= 1 \\
k^{SS} &= \left[\frac{(1-\alpha)A}{\frac{\gamma}{\beta}-1+\delta}\right]^{1/\alpha} \times N = 4.97 \\
y^{SS} &= N^{SS}A(k/N)^{1-\alpha} = 0.58 \\
i^{SS} &= k^{SS}(\gamma - 1 + \delta) = 0.144 \\
c^{SS} &= y^{SS} - i^{SS} = 0.4395 \\
\lambda^{SS} &= 1/c^{SS} = 2.27 \\
w^{SS} &= \theta(L^{SS})^{-\eta} = 2.88 \times 0.8^{-1} = 2.304
\end{aligned}$$

- Computation of the Solow residual

  Build a capital stock recursively from investment series, using δ. Issue of initial condition k_{1960Q1}.Set $k_{1960Q1} = k_{SS}$.

  Observe N_t

  Compute

$$\log(SR_t) = \log(GDP_t) - 0.66\log(N_t) - 0.33\log(k_t)$$

  Regress

  –
$$\log(SR_t) = \gamma t + \rho_a \log(SR_{t-1}) + \varepsilon_{a,t}$$

  ρ_a and σ_{ε_a} are central elements of the calibration

1.3.3 The model to be simulated

We now write the model as the system of FOC "behavioral" equations that relate to technology and preferences, for the specific functional form chosen through calibration.

The block
consumption: $u'_c(c_t, L_t) = \lambda_t$
leisure: $u'_L(c_t, L_t) = \omega_t$
labor: $\lambda_t A_t F'_N(k_t, N_t) = \omega_t$
capital accumulation: $\beta\lambda_{t+1}\left[A_t F'_k(k_{t+1}, N_{t+1}) + (1-\delta)\right] = \gamma\lambda_t$

becomes

$u'_c(c_t, L_t) = 1/c_t = \lambda_t$,
that log linearised (LL) is -log $\left(\frac{c_t}{c}\right) = \log\left(\frac{\lambda_t}{\lambda}\right)$ or $-c_t^{hat} = \lambda_t^{hat}$

$u'_L(c_t, L_t) = \theta L_t^{-\eta} = \omega_t$
or LL $\quad -\eta L_t^{hat} = \lambda_t^{hat} + w_t^{hat}$

$\lambda_t A_t F'_N(k_t, N_t) = 0.33 \times \lambda_t A_t k_t^{0.33} N_t^{-0.33} = \omega_t$
or LL $w_t^{hat} = A_t^{hat} + 0.33(k_t^{hat} - N_t^{hat})$

The capacity constraints, that are not dirrectly loglinear, are approximated by first order Taylor expansion
$(N)N_t^{hat} + (L)L_t^{hat} = 0$
$\left(\frac{c}{y}\right)c_t^{hat} + \left(\frac{i}{y}\right)i_t^{hat} = y_t^{hat} = A_t^{hat} + 0.66N_t^{hat} + 0.33k_t^{hat}$

likewise for the capital accumulation FOC.

$\beta\lambda_{t+1}\left[A_t F_k'(k_{t+1}, N_{t+1}) + (1-\delta)\right] = 0.99\lambda_{t+1}(0.33 A_t k_t^{-0.66} N_t^{0.66} + 0.975) = 1.004\lambda_t$

Linearise as exercise. Likewise for the law of motion of capital.

Altogether, we now have a system of equations that gives the dynamics of consumption, investment, the capital stock, output, labor, the real wage and the return on capital.

1.3.4 Simulation of the model in Dynare

Example to be simulated (taken from example 1 in the Dynare package). See Dynare User Guide for an introduction to the model.

Major difference with KR-HB is the functional form of the utility function that specifies a disutility of hours worked.

$$u(c_t, h_t) = \log(c_t) - \theta \frac{h_t^{1+\psi}}{1+\psi}$$

Program:

```
// Benoit Mojon, bmuniv@gmail.com
// stick to one shock to productivity
// almost exactly example 1 from Collard's guide to Dynare

periods 400;
var y, c, k, a, h, b;
varexo e, u;
parameters beta, rho, beta, alpha, delta, theta, psi, tau;
alpha = 0.36;
rho = 0.95;
tau = 0.025;
beta = 0.99;
delta = 0.025;
psi = 0;
theta = 2.95;
phi = 0.1;
model;

c*theta*h^(1+psi)=(1-alpha)*y;

k = beta*(((exp(b)*c)/(exp(b(+1))*c(+1))) *(exp(b(+1))*alpha*y(+1)+(1-
delta)*k));
```

```
y = exp(a)*(k(-1)^alpha)*(h^(1-alpha));

k = exp(b)*(y-c)+(1-delta)*k(-1);

a = rho*a(-1)+tau*b(-1) + e;

// Changed
b = 0.0;//tau*a(-1)+rho*b(-1) + u;
end;
```

1.3.5 Is the RBC model a good approximation of the true business cycle?

Approach: compare the "moments" of the time series from the simulated data and from the true data.

Momments= variance, relative variance, autocorrelation, cross-correlation, impulse responses.

Many of these directly available in Dynare output. Others we can compute from simulated time series.

Comment Table 1 of KR-HB

- Investment more variable than consumption
- Autocorrelation of variables
- Cross correlation: definition of Business cycle (Stock and Watson)
- Correlation of labor and output
- + response of hours to productivity shocks

Correlation of labor and output

- Prescott and RBC: labor supply responds to productivity shock [two stages of new theories]
- Modigliani: Recessions cannot be "vacation unemployement"
- One of the main controversies of Macroeconomics since Gali (AER-99) claim that labor's response to "acceleration in productivity is negative": hence productivity shocks are not dominating the business cycle.

  Ongoing fight that relates to identification assumption in reduced form models (VAR) of the data.

  Gali's result confirmed by Basu and Fernald

  But Fisher rescue the RBC by introducing another form of technology shocks: investment specific.

1.4 Synthesis of Lecture 1 to 3

1. Calibrate simple dynamic models to generate data. YWC
2. Compare properties of "artificial" and true data.YWC
3. Focus on the business cycle properties of the data. YWC

   However

4. No room for macroeconomic stabilisation policies in the RBC world.
5. No room for nominal variables.
6. Some of the dynamic properties of the model are not realistic

In the next lecture, we review the Christiano, Eichenbaum and Evans research agenda: to add the minimum set of frictions to the RBC model such that it comes close enough to the dynamic properties of macroeconomic time series.