# Interpretating Macroeconomic Time Series

# Sciences-Po M2 2009-2010

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**List of lectures**
*Lecture 1 and 2: Filtering and RBC calibration*
**Lecture 2 and 3: Toward the standard DSGE model**
Lecture 3 and 4 : VAR estimation
Lecture 4 and 5: VAR identification
Lecture 6 : Max Likelihood Estimation of a DSGE

## 1 Toward the standard DSGE

## Outline

1. **Introduction**
2. **Nominal rigidities**
3. **Real frictions**

## Reading list

Gali's monograph: Monetary Policy, Inflation and the Business cycle (Chap 2 and 3, slides are online)
Christiano, Eichenbaum and Evans (JPE, 2005)
Klenow and Malin (New Handbook of Monetary Economics)

## Exercise 1 for December 7

Compare IRFs to illustrate :

1. the effect of habit formation
2. the effect of price indexation
3. Taylor contracts / Calvo

## 1.1 Introduction

### 1.1.1 Shortcomings of the Plain Vanilla RBC model

- all real
- dynamics is largely endogenous
- dynamics at odd with "widely agreed" stylised facts: in particular, monetary policy has long and variable lags; the response of inflation takes time.
- as prices are always equating supply and demand, any increase in the money supply should immediately be reflected in higher prices with no impact on output

Let's take a RBC model with **prices and money but not capital** (Gali, Chapter 2)

Preferences:

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\psi}}{1+\psi}$$

Households FOC (log linearised)

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{1}$$

[1]

$$c_t = E_t\left[c_{t+1}\right] - 1/\sigma(i_t - E_t\left[\pi_{t+1}\right] - \rho) \tag{2}$$

$$m_t - p_t = y_t - \eta i_t \tag{3}$$

Technology:

$$\begin{aligned} Y_t &= A N_t^{1-\alpha} \\ y_t &= a_t + (1-\alpha) n_t \end{aligned} \tag{4}$$

Profit maximisation condition

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha) \tag{5}$$

Market equilibrium

[1] This comes from the following argument. Around the optimum, a variation of consumption needs to be "paid for" through more work or:
$U_{c,t} dC_t + U_{n,t} dN_t = 0$, while, at the same time, satisfying the budget constraint $P_t dC_t = W_t dN_t$.Combining both yields -$U_{n,t}/U_{c,t} = W_t/P_t$.With our assumption on the functional form of the Utility function, we have $C_t^{\sigma} N_t^{\varphi} = W_t/P_t$.

$$y_t = c_t \tag{6}$$

Money supply rule, e.g.

$$i_t = 1.5(\pi_t - \pi^*) + \varepsilon_{i,t} \tag{7}$$

Altogether, we can simulate this linear model with 7 unknown: $w, p, y, n, c, i$ and $m$

#### 1.1.2 Are prices Sticky of flexible?

Klenow and Malin survey: Most prices in the economy are sticky.

What do sticky price do to the variables dynamics?

We need a model to answer this question

Introducing sticky prices in an otherwise standard model (following Gali's chapter 3)

#### 1.1.3 The NK Philips curve

see Gali and the class of Aurelien for a detailed exposition of its derivation.

Same model as above except that:

- Producers have a pricing power on their market: monopolistic competition

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

- Ad hoc nominal rigidity: price stickiness à la Calvo (1983): each producer has a $(1-\theta)$ probability of being able to reset its price at each period

In this conditions, the profit maximisation of the producers that can adjust their price is given by

$$\max / P_t^* \sum \theta^k E_t \left\{ Q_{t,t+k} (P_t^* Y_{t+k} - \Psi_{t+k}(Y_{t+k})) \right\}$$

subject to

$$\begin{aligned} Y_{t+k} &= \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k} \\ Q_{t,t+k} &\approxeq \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) \end{aligned}$$

FOC

$$\sum \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k} (P_t^* - \frac{\epsilon}{\epsilon - 1} \psi_{t+k}) \right\} = 0$$

with $\psi_{t+k} = \Psi'_{t+k}(Y_{t+k})$

Producers maximise their profits for as long as they will be stuck at the price $P_t^*$, using the household discount factor.

Once log linearised:

$$p_t^* = \log(\frac{\epsilon}{\epsilon - 1}) + (1 - \beta\theta) \sum (\beta\theta)^k E_t \left[ mc_{t+k} + p_{t+k} \right]$$

in recursive form

$$p_t^* = \beta\theta E_t \left[ p_{t+1}^* \right] + (1 - \beta\theta) \left[ mc_t + p_t \right]$$

with

$$\begin{aligned} p_t &= \theta p_{t-1} + (1 - \theta) p_t^* \\ \pi_t &= (1 - \theta)(p_t^* - p_{t-1}) \\ \pi_{t+1} &= (1 - \theta)(p_{t+1}^* - p_t) \end{aligned}$$

$$\begin{aligned} \pi_t &= (1 - \theta)(\beta\theta E_t \left[ p_{t+1}^* \right] + (1 - \beta\theta) \left[ mc_t + p_t \right] - p_{t-1}) \\ &= (1 - \theta)(\beta\theta E_t \left[ p_{t+1}^* - p_t \right] + \beta\theta p_t + (1 - \beta\theta) \left[ mc_t + p_t \right] - p_{t-1}) \\ &= \beta\theta E_t \left[ \pi_{t+1} \right] + (1 - \theta)(p_t - p_{t-1} + (1 - \beta\theta) \left[ mc_t \right]) \\ &= \beta\theta E_t \left[ \pi_{t+1} \right] + (1 - \theta)(\pi_t + (1 - \beta\theta) \left[ mc_t \right]) \\ \pi_t &= \beta E_t \left[ \pi_{t+1} \right] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \left[ mc_t \right] \end{aligned}$$

From the marginal cost to output

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1 - \alpha) \end{aligned}$$

gets the NK Ph Curve

$$\pi_t = \beta E_t \left[ \pi_{t+1} \right] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} (\sigma + \frac{\varphi + \alpha}{1 - \alpha})(y_t - ybar) \tag{8}$$

inflation depends on future inflation and tensions on demand. The impact of tensions on demand on inflation depends in a simple/complex way of the strutural parameters of the model: the price stickiness (or Calvo parameter), the curvature of the utility function, the disutility of labor, the labor share,...

## 1.2 Alternative forms of nominal rigidities

Taylor stagered contracts
State dependant pricing
Rational inatention a la Mankiw and Reis
Rational inatention à la Sims, Markowiak and Wiederholt

## 1.3 Additional frictions in CEE

- Habit formation

$$u(c_t - hc_{t-1}, n_t) = \frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\psi}}{1+\psi}$$

  somewhat ad hoc. used in finance, buys hump shapes in IRF

  Experiment using the simulation program

- Indexation: buys "inflation persistence", see Fuhrer New Monetary Economics Handbook chapter
- Wage stickiness
- Variables capital utilisation ratio
- Investment adjustment costs

### 1.3.1 Simulation program

```
// Aurelien Poissonnier (aurelien.poissonnier@insee.fr)
// octobre 2009
var y c w n r pie lambda e_r e_a e_g e_p;
varexo    ee_r ee_a ee_g ee_p;
parameters r_pi r_y sig_c h alpha c_cons beta xi rho_r rho_a rho_g rho_p;
r_pi=1.5;
r_y=0.3;
sig_c=2;
alpha=0.3;
c_cons=0.6;
beta=0.99;
h=0.95;
xi=0.75;
rho_r=0.4;
rho_a=0.4;
rho_p=0.4;
rho_g=0.4;
model(linear);
// Taylor rule
```

```
r=r_pi*pie+r_y*(y-y(-1))+e_r;
//marginal utility
lambda=-sig_c/(1-h)*(c-h*c(-1));
//production function
y=(1-alpha)*n+e_a;
//market clearing
y= e_g+c_cons*c;
//euler equation
lambda(+1)-lambda+r-pie(+1)=0;
// phillips curve
xi*pie=xi*beta*pie(+1)+(1-beta*xi)*(1-xi)*(w+alpha*n+e_p);
// wage setting
w=e_a;
// shocks AR(1)
e_a=rho_a*e_a(-1)+ee_a;
e_r=rho_r*e_r(-1)+ee_r;
e_g=rho_g*e_g(-1)+ee_g;
e_p=rho_p*e_p(-1)+ee_p;
end;
shocks;
var ee_a; stderr 1;
var ee_r; stderr 1;
var ee_g; stderr 1;
var ee_p; stderr 1;
end;
steady;
check;
stoch_simul(irf=10,periods=1000) y c n r pie w;
```