Ecole Polytechnique - HEC

Master Quantitative Economics and Finance 2013-2014

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ECO 569
MONETARY ECONOMICS

LECTURE NOTES

Lecture 1: Macroeconomic stabilization policies, 10 January
Lecture 2: The “pre crisis” dominant monetary policy doctrines, 17 January
Lecture 3: The transmission of monetary policy, 24 January
Lecture 4: The liquidity trap, 31 January
Lecture 5: Banks and the central bank, 7 February
Lecture 6: Financial cycles and crises, 14 February
Lecture 7: Financial regulation, 21 February
Lecture 8: The crisis of the euro, 7 March
Lecture 9: Wrap up + exam prepa, Thursday 13 March

0.0.1 General references used through out

Alan Blinder: "Sixteen questions and about 12 answers about monetary policy", available on the web

Pierpaolo Benigno: "New Keynesian Economics: An Aggregate Demand / Aggregate Supply view", NBER working paper

David Romer: "Traditional Keynesian Theories of Fluctuations", Chapter 5 in Advance Macroeconomics, MacGraw Hill.
1 Lecture: Macroeconomic stabilization policies

1.1 Introduction

1.1.1 What is macroeconomics? Aggregate output, labor, capital, productivity
consumption, investment, exports, imports
government budget (expenditure) and taxes, fiscal policy
intertemporal allocation/finance: savings, assets, asset prices
money, interest rates, exchange rates, monetary policy

1.1.2 3 Major Crises and the design of macroeconomics policies

- The Great Depression of the 1930s and Keynesian macroeconomics
- The Great Inflation of the 1970s and the Rational Expectations revolution
- The Great recession of 2009 as showed in a few pictures bellow

1. Its origin in excess leverage
2. Fiscal policy response (let deficit run and public debt grow)
3. Conventional monetary policy response (drop "policy interest rates")
4. Unconventional monetary policy response (increase the quantity of central bank money issued)
Figure 1: Household debt over GDP
Figure 2: NFC debt over GDP

Figure 3: The worst post WW2 recession: German GDP growth rate
Figure 4: The worst post WW2 recession: French GDP

Figure 5: Public sector debt over GDP
Figure 6: EA money market interest rates 2007-2013

Figure 7: CB balance sheet / GDP
Today’s Outline

1. From IS-LM to AS-AD
2. Nominal rigidities as the most widely used form of money non-neutrality
3. The dynamic budget constraint of the government

1.2 From IS-LM to AS-AD (Benigno)

Preliminary remarks

The objective of today’s class is to re-visit and review macroeconomics as they first developed in the aftermath of the Great Depression of the 1930s. J.M. Keynes published the General Theory in 1936 in order to propose a positive theory of the Great Depression. Why had the economy drifted in a prolonged slump with so high unemployment (25% in the US). (Some of T)he main intuitions of the General Theory were modelled by Hicks in 1944 into the IS-LM model that has become the most widely used macro model in textbooks since WW II. We review this model for 2 reasons:

- It is the most widely used model of macro. Elements of common knowledge/language/intuition/theory need to be learned;
- Its simplicity for a first pass on the key questions of macro: the neutrality of money; the qualitative effects of monetary and fiscal policy

The main drawback of this approach to macro is that it is static. This approach was in particular overthrown by the Rational Expectations paradigm in the 1970’s which focused on the forward looking nature of the intra and intertemporal allocation of resources.

Some consider comforting that dynamic macro are somewhat consistent with endogenous "behavioral" decisions of a maximising "individual", most of the time a "representative individual" which decisions are "aggregated" to get to the macro level. Most of the class presents the core of dynamic macro as it has developed since the RE revolution.

Some however doubt that dynamic macro has brought "progress" in the provision of models (theories) that are less rejected by the data than the main intuitions from static Keynesian economics (see the discussion in the introduction and the literature review of Benigno).

The IS-LM model is a model of aggregate demand, considering that prices are rigid. By definition, a transaction involves a price and a quantity. If prices are rigid (or fixed) changes in transactions involve a change in quantity. Stated otherwise, a change in demand will be reflected in a change of (supplied) output.
1.2.1 IS-LM

Agents hold either money or bonds. The demand for real money balances increases with demand (consumption, investment) for transactional purposes and decreases with the interest rate, which is the opportunity cost of holding money. The supply of money is fixed by the central bank.

The LM curve is

\[ \frac{M}{P} = F(Y, i), \text{ with } F_Y > 0 \text{ and } F_i < 0 \]

For a given \( \frac{M}{P} \), higher \( Y \) implies a higher \( i \) for the equation to hold. Hence this LM curve is upward sloping in the \((Y, i)\) plan.

The IS curve just states that demand (durable, investment goods, but also consumption will be lower for higher levels of the real interest rate:

\[ Y = F^*(Y, i - \Delta P, G, T), \text{ with } F_Y > 0 \text{ and } F_i < 0 \]

Hence the IS curve is downward sloping in the \((Y, i)\) plan. We therefore can get an equilibrium \((Y^E, i^E)\) at the cross of the IS and the LM curve for any price level.

Let us now assume a change (an increase) in the price level.

- lower real money balances requires a higher nominal interest rate for the money market to clear.
- this higher interest rate implies a lower demand.
- >> hence the AD curve is downward sloping.

1.2.2 "Micro-founded" based AD

Assumption 1: The representative household maximise a utility that increases in consumption and decreases in the time spent working

\[ U_H = u(C) - v(L) + \beta[u(\bar{C}) - v(\bar{L})] \]

under the budget constraint

\[ PC + \frac{\bar{C} \bar{P}}{1 + \bar{i}} = WL + \frac{\bar{W} \bar{L}}{1 + \bar{i}} + T \]

\( \bar{C}, \bar{P}, \bar{W} \) and \( \bar{L} \) are period 2 consumption, price, wage and hours worked; \( \beta \) and \( i \) are the discount rate and the nominal interest rate. \( T \) is the net tax received or paid by households + firms’ profits.

Maximisation of utility under the budget constraint implies two intra-temporal and one inter-temporal first order conditions:

\[ \frac{v'(L)}{u'(C)} = \frac{W}{\bar{P}} \]
The real wage compensate exactly the marginal disutility of marginal hour worked scaled by the marginal utility of consumption.

Turning to allocation of consumption across periods, given by the Euler equation

\[
\frac{u'(C)}{u'(C)} = (1 + i) \frac{P}{P} = (1 + r)
\]

Assuming a standard isoelastic utility \( u(C) = C^{\frac{1}{1-\sigma^*}}, \) \( u'(C) = C^{-1/\sigma^*}, \) where \( \sigma^* \) is the intertemporal elasticity of consumption. Taking the log of the Euler equation for this functional form of the Euler equation, we get (lower case denotes logs of upper case)

\[ \bar{c} - c = \sigma^* (i - (\bar{p} - p)) + \sigma \ln \beta \]

This Euler equation is an IS curve, whereby current demand depends negatively on the real interest rate. It can be also seen as an aggregate demand equation that relates negatively current demand to the current prices (\textit{ceteris paribus} higher current prices increase the real interest rate).

\textbf{Remark:} The traditional text book derivation of the IS curve is postulated from intuition (higher interest rate increase the yield on saving and the cost of borrowing) rather than derived from a optimisation of the representative household utility in this two period framework. Benigno’s message is: if you feel more confortable with equations that are derived from optimizing behavior, you can easily relate the traditional Keynesian model to it. Don’t criticise these models on this basis. "All models are wrong, some are useful". Which one of the two models is the most useful?

\subsection{1.2.3 Adding public expenses}

\[ Y = C + G \]
\[ \bar{Y} = \bar{C} + \bar{G} \]

In logs

\[ y = s_c c + g \]
\[ \bar{y} = s_c \bar{c} + \bar{g} \]

before substituting into the Euler equation for consumption to get an AD expression

\[ y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p)] + \sigma \ln \beta \]

where \( \sigma = s_c \sigma^*. \)

This AD expression has a negative slope between \( y \) and \( p. \)
1.3 Aggregate supply with nominal rigidities

Prices need to be rigid for non-neutrality of shifts in demand.

1.3.1 Are prices Sticky or flexible?

Look at the evidence on price rigidities gathered in Klenow and Malin (Monetary Economics Handbook survey, on my webpage):

Most prices in the economy are sticky.

1.3.2 Modeling price setting decisions (Benigno)

Assuming that price rigidities are non-trivial, a convenient way to model price rigidities is to consider that

1. price setters are subject to monopolistic competition (they produce differentiated goods that are imperfectly substitutable)

2. only a fraction $1 - \alpha$ of producers are able to adjust their price at each period (this is known as the Calvo model of price rigidities). A fraction $\alpha$ of producers cannot change their price (for menu cost reasons) and set it to a predetermined level $P^e$.

Producer $j$ aims at maximising profits

$$\Pi(j) = P(j)Y(j) - WL(j)$$

with demand for good $j$ being

$$Y(j) = \left(\frac{P(j)}{P}\right)^{-\theta}(C + G)$$

and production being

$$Y = AL$$

The profit maximising price is a mark up over marginal cost with the mark up being scaled by the elasticity of substitutions between goods $\theta$:

$$P^o(j) = \frac{\theta}{\theta - 1} \frac{W}{A} = \frac{(1 + \mu)W}{A}$$

$$\frac{P^o(j)}{P} = \frac{1 + \mu W}{A \cdot P} = \frac{1 + \mu v'(L)}{A \cdot u'(C)}$$

In the case all firms could adjust prices, we would have the "natural" production level at its socially optimal level given by:
\[
\frac{v'(L)}{u'(C)} = \frac{W}{P} \iff \frac{v'(Y_n/A)}{u'(Y_n - G)} = \frac{W}{P} = \frac{A}{1 + \mu}
\]

with the functional forms \(u(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma}\) and \(v(L) = L^{1+\eta}/(1+\eta)\); we get

\[
\frac{(Y_n/A)^\eta}{(Y_n - G)^{-\eta}} = \frac{A}{1 + \mu}
\]

in logs

\[
\eta(y_n - a) + \frac{1}{\sigma^*}(y_n + \log(1 - \frac{G}{Y_n})) = a - \log(1 + \mu)
\]

\[
y_n \approx \frac{1}{\eta + 1/\sigma} a + \frac{1}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma}^\mu
\]

Now, let’s consider the effect of nominal rigidities:

1 - \(\alpha\) of producers who can reset optimal prices:

\[
\frac{P^\circ}{P} = \frac{1 + \mu v'(L)}{A u'(C)} = \left(\frac{(Y_n/A)^\eta}{(Y_n - G)^{-\eta}}\right)^{-1} \frac{v'(L)}{u'(C)} = \left(\frac{(Y_n/A)^\eta}{(Y_n - G)^{-\eta}}\right)^{-1} \frac{(Y/A)^\eta}{(Y - G)^{-\eta}}
\]

In logs

\[
p^\circ - p = (\eta + \frac{1}{\sigma})(y - y_n)
\]

To get to the general price level, consider the \(\alpha\) producers who did not get the opportunity to reset prices

\[
p = \alpha p^e + (1 - \alpha)p^\circ
\]

We end up to a Philips curve equation

\[
p - p^e = \kappa(y - y_n) \text{ with } \kappa = \frac{1 - \alpha}{\alpha}(\eta + 1/\sigma)
\]

where the slope of the price with respect to the "output gap" increases with "the disutility of labor", the intertemporal elasticity of substitution and the proportion of firms that can reset prices to adapt to shifts in demand.
1.4 The AD-AS model in action (Benigno)

We now have a 3 equations model with output, natural/flex-price output, prices and the nominal interest rate and we can do comparative static analyses of shocks to the economy.

\[ p - p^e = \kappa(y - y_n) \]

\[ y = \bar{y} + (g - \bar{g}) - \sigma[i - (\bar{p} - p)] + \sigma \ln \beta \]

\[ y_n \simeq \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu \]
$p - p^e = \kappa(y - y_n)$

$y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p)] + \sigma \ln \beta$

$y_n \approx \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu$

Figure 8: Positive supply shock in Benigno’s model
Figure 9: Positive supply shock and monetary policy stabilisation in Benigno’s model

\[ p - p^e = \kappa(y - y_n) \]

\[ y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p)] + \sigma \ln \beta \]

\[ y_n \approx \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu \]
Figure 10: Permanent positive supply shock in Benigno’s model

\[ p - p^e = \kappa (y - y_n) \]

\[ y = \bar{y} + (g - \bar{g}) - \sigma \left[ i - (\bar{p} - p) \right] + \sigma \ln \beta \]

\[ y_n \approx \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu \]
Figure 11: Trade off between price and output stabilisation following an increase in mark up, Benigno’s model

\[ p - p^e = \kappa(y - y_n) \]

\[ y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p)] + \sigma \ln \beta \]

\[ y_n \simeq \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu \]
1.5 Fiscal policy and public debt

1.5.1 Outline

1. The budget constraint of the government
2. The Keynesian fiscal policy multiplier
3. Ricardian equivalence
4. Fiscal policy in an OLG model
5. A model of sovereign debt crisis

1.5.2 The dynamic budget constraint of the government

Given the current situation in Europe and the US, it seems appropriate to review the economic debate on fiscal policy, the role of fiscal deficits and the dynamic of public debt. A few data to recall the current situation.

A reminder on (public) debt dynamic

\[ B_t = (1 + R_{t-1})B_{t-1} + G_t - T_t - (M_t - M_{t-1}) \]

Most public discussions are actually about debts to GDP ratios

\[ \frac{B_t}{P_t Y_t} = (1 + R_{t-1}) \frac{P_{t-1}Y_{t-1}}{P_t Y_t} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} + \frac{D_t}{P_t Y_t} \Delta M_t \]

\[ b_t = \frac{1 + R_{t-1}}{1 + \pi_t + g_t} b_{t-1} + d_t - \Delta m_t \]

Preview on central banks interventions and multiple equilibria

Take the case of Italy and Germany, taking \( d_t^i \) as the primary surplus that stabilises \( b_t \):

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<th>June 11</th>
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<th>August 11</th>
<th>January 13</th>
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<td>( R_t )</td>
<td>5%</td>
<td>6%</td>
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<td>4%</td>
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<td>( b_t )</td>
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<td>( \pi_t )</td>
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<td>( g_t )</td>
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<td>( \frac{1 + R_{t-1}}{1 + \pi_t + g_t} )</td>
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<tr>
<td>( d_t^i )</td>
<td>2.4%</td>
<td>3.6%</td>
<td>2.4%</td>
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</tr>
</tbody>
</table>
1.5.3 What is fiscal policy: public expenditure and income: The macro side of fiscal policy

1.5.4 The Keynesian multiplier

This approach has been heavily criticized since the 1970s. Many consider it is nonsense. It was however taught as one of the entry points of macro since WW2.

Take the accounting identity between output/income and expenditures, assuming prices are fixed:

\[
Y_t = C_t + I_t + G_t
\]

\[
C_t = c(Y_t - T_t)
\]

\[
I_t = aY_t - br_t + \bar{I}
\]

Solving this equation for \( Y_t \)

\[
Y_t = \frac{\bar{I} + G_t - br_t - cT_t}{1 - c - a}
\]

The "dynamic effects" of the multiplier is due to : the extra output/income that comes from public expenditure will be consume or invested.

Extension: Assume now that fiscal expenditure (e.g. transfers to unemployed), decreases with output, while fiscal income is proportional to output (e.g. income tax, corporate tax on profits,...)

\[
G_t = \bar{G} - gY_t
\]

\[
T_t = \bar{T} + \tau Y_t
\]
We now get
\[ Y_t = \frac{\bar{I} + \bar{G} - br_t - c\bar{F}}{1 + g + c\tau - c - a} \]
These effects reduce the size of the "Keynesian" multiplier.

1.5.5 Fiscal policy in a dynamic model: Ricardian equivalence

Let us build a dynamic model with firms that produce, a household that consume and a government.

**The firms**

\[ Y_t = F(K_t, L_t) \]

firms are price takers on the market for production inputs. Profit maximisation yields

\[ F'_K(K_t, L_t) = 1 + r_t \]
\[ F'_L(K_t, L_t) = w_t \]

**The households** The derive utility from a time separable concave utility function and can accumulate assets. The size of the population is constant and fixed to \( L \).

\[ c_t + A_{t+1} + T_t \leq w_t L + (1 + r_t)A_t \]

We assume a standard transversality condition and obtain the standard Euler for the inter-temporal allocation of consumption:

\[ u'(c_t) = \beta (1 + r_{t+1})u'(c_{t+1}) \]

**The government**

\[ G_t + (1 + r_t)B_t = T_t + B_{t+1} \]

**Markets equilibria**

- Financial markets
  \[ A_{t+1} = B_{t+1} + K_{t+1} \]
- Goods markets
  \[ c_t + G_t + K_{t+1} = Y_t \]
- Labor markets
  \[ L_t = L \]
The long run equilibrium  The steady state consumption condition set the steady state interest rate and hence the level of the capital stock

\[ u'(c_t) = \beta(1 + r)u'(c_{t+1}) \]
\[ 1 + r = \frac{1}{\beta} \]
\[ F'_K(K_t, L_t) = \frac{1}{\beta} \]

If \( 1 + r > \frac{1}{\beta} \), savings increase the capital stock until \( F'_K(K_t, L_t) = \frac{1}{\beta} \). Hence, the steady state capital stock is pinned down by technology and preferences:

\[ F'_K(K^*, L) = \frac{1}{\beta} \]

From the goods' market, we get

\[ c = F(K^*, L) - G - K^* \]

so that any increase in government consumption reduces consumption by as much.

Summing the budget constraints of households

\[ c_t + A_{t+1} + T_t = w_t L + (1 + r_t)A_t \]
\[ c_{t+1} + A_{t+2} + T_{t+1} = w_{t+1} L + (1 + r_{t+1})A_{t+1} \]

we get

\[ \sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^t} + \lim_{t \to \infty} \frac{A_t}{(1 + r)^t} = \sum_{t=0}^{\infty} \frac{w_t L}{(1 + r)^t} - \sum_{t=0}^{\infty} \frac{T_t}{(1 + r)^t} \]

Provided the government respects its budget constraint, i.e. it pays back its debts, its long term budget constraint is simply

\[ \sum_{t=0}^{\infty} \frac{T_t}{(1 + r)^t} = \sum_{t=0}^{\infty} \frac{G_t}{(1 + r)^t} \]

hence

\[ \sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^t} + \lim_{t \to \infty} \frac{A_t}{(1 + r)^t} = \sum_{t=0}^{\infty} \frac{w_t L}{(1 + r)^t} - \sum_{t=0}^{\infty} \frac{G_t}{(1 + r)^t} \]

The financing of governement expenditure through debt or taxes is irrelevant for the long term wealth of households.

This is a an extreme result that is nevertheless a benchmark.
1.5.6 Insights from the current public debt outlook

- What aspects of the previous model do not hold for Italy, Spain, Greece?
- Permanent income model of consumption choices

1.5.7 Public debt in an OLG set up (Romer, chap 2 part B)

Overlapping generation models is one of the oldest models to analyse the intertemporal allocation of resources. It brings interesting intuition to study fiscal policy and public debt in part because it allows transfers across "generations".

Assumptions of the model:

- Each generation lives for two periods and at each point in time a young and an old generation overlap.
- The utility of one generation is assumed to be

\[ U_t = \frac{C_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_{2t+1}^{1-\theta}}{1-\theta} \]

- Only the young generation supply labor to produce

\[
Y_t = F(K_t, A_t L_t) \\
A_t = (1+g)A_{t-1}; \text{exogenous labor prod. aug; technology} \\
r_t = f'(k_t) \\
w_t = f(k_t) - k_t f'(k_t)
\]

- The consumption of the old amounts what they saved when young

\[
C_{2t+1} = (1+r_{t+1}) (w_t A_t - C_{1t}) \\
C_{1t} + \frac{C_{2t+1}}{1+r_{t+1}} = w_t A_t; \text{ the budget constraint of a generation}
\]

Model resolution  Maximising \( U_t \) given this budget constraint yields a typical intertemporal allocation of consumption

\[
\frac{C_{2t+1}}{C_{1t}} = \left( \frac{1+r_{t+1}}{1+\rho} \right)^{1/\theta}
\]

Substituting back into the budget constraint, we have
\[ C_{1t}(1 + \frac{(1 + r_{t+1})^{1-g}}{(1 + \rho)^{1/g}}) = A_t w_t \]

\[ C_{1t} = \frac{(1 + \rho)^{1/g}}{(1 + \rho)^{1/g} + (1 + r_{t+1})^{1-g}} A_t w_t \]

\[ C_{1t} = (1 - s(r_{t+1})) A_t w_t \]

Early consumption of the young (savings) of the youngs decrease (increase) with the level of the return on capital.

The dynamic of capital in the model is

\[
\begin{align*}
K_{t+1} &= s(r_{t+1}) L_t A_t w_t \\
\frac{K_{t+1}}{L_{t+1} A_{t+1}} &= s(r_{t+1}) w_t \frac{L_t A_t}{L_{t+1} A_{t+1}} \\
k_{t+1} &= \frac{s(r_{t+1}) w_t}{(1 + g)(1 + n)} \\
k_{t+1} &= \frac{s(f'(k_t))(f(k_t) - k_t f'(k_t))}{(1 + g)(1 + n)}
\end{align*}
\]

Introducing government

- Assume first that all government expenditure are financed by taxes.

\[
k_{t+1} = \frac{s(f'(k_t))(f(k_t) - k_t f'(k_t) - G_t)}{(1 + g)(1 + n)}
\]

Hence \( G_t \) reduces the income base of savings and the rhythm of capital accumulation.

- Assume second that government expenditures are financed by taxes or by bonds \( b_{t+1} = \frac{B_{t+1}}{A_t L_t} \).

\[
k_{t+1} = \frac{s(f'(k_t))(f(k_t) - k_t f'(k_t) - T_t) + b_{t+1}}{(1 + g)(1 + n)}
\]

In this case, some of the government expenditure is financed by future taxes, i.e. future generations.

- Discussion of the model

  Horizon of debt issuance and life expectancy

**Remark:** we will revisit the debate on the fiscal multiplier in Lecture 4 on the liquidity trap.
1.6 Public debt sustainability and sovereign debt crises

1.6.1 The post war dynamic of the US public debt: Hall and Sargent

From 1945 to 1974, the US government debt/GDP declined from 97.2% to 16.9% of GDP. From 1993 to 2001, it fell from 48.2% to 19.9%. Given the steep increase in public debt following the great recession and the nervosity of financial markets / sustainability of public debt, it is useful to look into the determinants of such massive downward adjustments in public debts.

This is exactly the purpose of Hall and Sargent’s paper. In the paper, there is an extended analysis of debt maturity and the interest rate risk born by the US treasury and debt holders. We neglect this part of the article here and focus instead on the four major determinants of debt/GDP which are: interest payments, inflation, growth and the primary surplus.

Recall

\[
\frac{B_t}{Y_t} = (1 + R_{t-1}) \frac{P_{t-1}Y_{t-1}}{P_tY_t} \frac{B_{t-1}}{Y_{t-1}} + \frac{D_t}{P_tY_t}
\]

with now \(B_t\) and \(D_t\) are real,

\[
\frac{B_t}{Y_t} = (1 + R_{t-1} - \pi_t - g_t) \frac{B_{t-1}}{Y_{t-1}} + \frac{D_t}{Y_t}
\]

Now let consider that \(B_t\) is the sum of nominal \(\tilde{B}_t\) and inflation index bonds \(\tilde{B}_t\) and that each type of bond is decomposed into bonds of different maturities. We can rewrite the above equation as

\[
\frac{\tilde{B}_t + B_t}{Y_t} = \sum_{j=1}^{n} \tilde{r}_{t-j,t} \frac{\tilde{B}_{t-j}^{j}}{Y_{t-j}} - (\pi_t + g_t) \frac{\tilde{B}_{t-1}^{j}}{Y_{t-1}}
\]

\[
+ \sum_{j=1}^{n} \tilde{r}_{t-j,t} \frac{B_{t-j}^{j}}{Y_{t-1}} - (g_t) \frac{B_{t-1}^{j}}{Y_{t-1}} + \frac{D_t}{Y_t} + \frac{\tilde{B}_{t-1} + B_{t-1}}{Y_{t-1}}
\]

Iterating this equation from a given point in time, we get

\[
\frac{\tilde{B}_t + B_t}{Y_t} - \frac{\tilde{B}_{t-s} + B_{t-s}}{Y_{t-s}} = \sum_{s=0}^{\tau-1} \left[ \sum_{j=1}^{n} \left( \tilde{r}_{t-j,t} - (\pi_{t-s-1} + g_t) \tilde{B}_{t-s-1}^{j} \right) \frac{\tilde{B}_{t-s-1}}{Y_{t-s-1}} \right]
\]

\[
+ \sum_{j=1}^{n} \left( \tilde{r}_{t-j,t} - g_{t-s-1} \right) \frac{B_{t-s-1}^{j}}{Y_{t-s-1}} + \frac{D_{t-s}}{Y_{t-s}}
\]

Insert figure 7 of Hall and Sargent

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1.6.2 A (simple) model of debt crisis (Romer, Advance Macroeconomics section 11.10)

Assume a government need to roll over its debt $B_0$, has no income tax this period and it expects to levy taxe income $T_1$ next period. $T_1$ is however uncertain and its cumulative distribution is given by $F(T_1)$. The government pays an interest $R - 1$.

We assume that if $T_1 < RB_0$, the government default on its debt and the return to investors is nil.

Assuming that investors are risk neutral, they want that the return on lending to this government compensate for the default risk:

$$\pi 0 + (1 - \pi)R = \bar{R}$$

where $\pi$ is the probability that the government will default.

Notice first that the level of interest rate will therefore have an impact on the default probability:

$$\pi = \frac{R - \bar{R}}{R}$$

Turning to the risk on fiscal income, we have also that the government will default when $T_1 < RB_0$ :

$$\pi = F(RB_0)$$

If the density distribution of $T_1$ has a bell shape, $F(T_1)$ has a $S$ shape.

In this case, we get multiple equilibria (3) and these equilibria are unstable.
Figure 12: Multiple equilibria on public debt
2 Lecture: The “pre crisis” dominant monetary policy doctrines

2.0.3 Outline

1. Introduction: some fundamentals about money, and the role of monetary policy
   An overview of monetary institutions and recent (post 1970) major events
2. From the Great inflation to central bank independance
3. Inflation stability

2.1 From the Great inflation to central bank independance

2.1.1 Historical evidence

Stylized facts (Ciccarelli and Mojon)

1. 70% of the variance of 22 OECD countries inflation is common (driven by a single factor)
2. Even higher proportion at lower frequency

Change in institutions

1. The 1970s inflation
   Productivity slowdown, oil price shock and monetary policy mistakes
2. The 1980s disinflation
   • 1979 ERM in Europe (France, Italy) "import the Bundesbank credibility"
   • 1980 Monetarism implemented in the US (Volker + Reagan) and in the UK (Thatcher)
3. The 1990’s disinflation
   • The Maastricht treaty
   • The spreading of inflation targeting (NZ, in 1989; Canada, UK, Sweden in 1992; many more since)
<table>
<thead>
<tr>
<th>Country</th>
<th>Average</th>
<th>OECD</th>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greece</td>
<td>0.42</td>
<td>0.66</td>
<td>0.38</td>
<td>0.15</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.47</td>
<td>0.24</td>
<td>0.41</td>
<td>0.16</td>
</tr>
<tr>
<td>Japan</td>
<td>0.56</td>
<td>0.30</td>
<td>0.52</td>
<td>0.14</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.58</td>
<td>0.26</td>
<td>0.58</td>
<td>0.19</td>
</tr>
<tr>
<td>Germany</td>
<td>0.60</td>
<td>0.33</td>
<td>0.57</td>
<td>0.16</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.63</td>
<td>0.62</td>
<td>0.61</td>
<td>0.14</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.65</td>
<td>0.65</td>
<td>0.63</td>
<td>0.09</td>
</tr>
<tr>
<td>United States</td>
<td>0.68</td>
<td>0.72</td>
<td>0.66</td>
<td>0.02</td>
</tr>
<tr>
<td>Norway</td>
<td>0.70</td>
<td>0.58</td>
<td>0.68</td>
<td>0.03</td>
</tr>
<tr>
<td>Australia</td>
<td>0.73</td>
<td>0.70</td>
<td>0.71</td>
<td>0.06</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.73</td>
<td>0.54</td>
<td>0.71</td>
<td>0.00</td>
</tr>
<tr>
<td>Austria</td>
<td>0.74</td>
<td>0.40</td>
<td>0.72</td>
<td>0.12</td>
</tr>
<tr>
<td>Spain</td>
<td>0.75</td>
<td>0.58</td>
<td>0.74</td>
<td>0.03</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.76</td>
<td>0.64</td>
<td>0.71</td>
<td>0.02</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.76</td>
<td>0.47</td>
<td>0.77</td>
<td>0.02</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.83</td>
<td>0.66</td>
<td>0.81</td>
<td>0.00</td>
</tr>
<tr>
<td>Finland</td>
<td>0.83</td>
<td>0.58</td>
<td>0.81</td>
<td>0.01</td>
</tr>
<tr>
<td>Canada</td>
<td>0.83</td>
<td>0.75</td>
<td>0.81</td>
<td>0.04</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.84</td>
<td>0.56</td>
<td>0.83</td>
<td>0.03</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.85</td>
<td>0.60</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>Italy</td>
<td>0.86</td>
<td>0.81</td>
<td>0.86</td>
<td>0.03</td>
</tr>
<tr>
<td>France</td>
<td>0.89</td>
<td>0.73</td>
<td>0.90</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean</td>
<td>0.71</td>
<td>0.56</td>
<td>0.69</td>
<td>0.06</td>
</tr>
<tr>
<td>Median</td>
<td>0.74</td>
<td>0.59</td>
<td>0.71</td>
<td>0.03</td>
</tr>
<tr>
<td>Euro area</td>
<td>0.95</td>
<td>0.76</td>
<td>0.95</td>
<td>0.00</td>
</tr>
</tbody>
</table>
2.1.2 The time inconsistency model of inflation

The model/theory underlying the change in institutions.

Basic idea of Kindland-Prescott (JPE-1977) and Barro-Gordon (JME-1983) is that monetary authorities / governments cannot try to exploit the Philips curve to attain a higher level of output thanks to lower real interest rates through higher inflation.

2.1.3 A model of time inconsistency (Romer, section 4 of monetary policy chapter)

Assume the following supply curve:

\[ y = \bar{y} + b(\pi - \pi^e) \]

and a quadratic social welfare function:

\[ L = \frac{1}{2}(y - y^*)^2 + \frac{1}{2}a(\pi - \pi^e)^2 \]

\[ y^* > \bar{y} \]

with \( \pi \), the policy instrument of the authorities.

Taking \( \pi^e \) as given, we can substitute the supply curve into the objective function and derive the first order condition with respect to \( \pi \):
$$L = \frac{1}{2}(\bar{y} + b(\pi - \pi^*) - y^*)^2 + \frac{1}{2}a(\pi - \pi^*)^2 \quad \pi = \pi^* + \frac{b}{a + b^2}(y^* - \bar{y}) + \frac{b^2}{a + b^2}(\pi^* - \pi^*)$$
\[b(\bar{y} + b(\pi - \pi^*) - y^*) + a(\pi - \pi^*) = 0\]

The only equilibrium, however is when $\pi^* = \pi$. Hence:

$$\pi^* = \pi^* + \frac{b}{a}(y^* - \bar{y})$$
$$y = \bar{y}$$

**What solution to this inflation bias?**

Rogoff (1985) famously proposed to assign monetary policy to a delegate who is widely known to dislike inflation.

This can be represented in the above model. This "conservative / inflation adverse" delegate would have the following preferences:

$$L_{conservative} = \frac{1}{2}(\bar{y} + b(\pi - \pi^*) - y^*)^2 + \frac{1}{2}a_{cons}(\pi - \pi^*)^2 \quad a_{cons} > a$$

Solving the model

$$\pi^* = \pi^* + \frac{b}{a_{cons}}(y^* - \bar{y}) < \pi^* + \frac{b}{a}(y^* - \bar{y})$$
$$y = \bar{y}$$

Central bank independance from "governments".

- Bundesbank
- New Zeland "job oppening for governor"

### 2.2 Inflation stability

Inflation has become remarkably stable through out the world since 1995. Why is that so?

#### 2.2.1 Globalisation?

The emergence of China is a huge positive supply shock.
2.2.2 Central bank independance and mandate

- Example, ECB
- See discussion in Ciccarelli and Mojon, www.benoitmojon.com/research

2.2.3 The cannonical model NK model of the inflation targeting sucess

See the science of monetary policy (Clarida, Gali and Gertler, 2000)

\[
\pi_t = E_t[\pi_{t+1}] + \kappa(y_t - y_n)
\]
\[
y_t = \beta E_t[y_{t+1}] - \sigma (i_t - E_t[\pi_{t+1}])
\]
\[
i_t = \rho i_{t-1} + (1 - \rho) \left( \bar{r} + \pi^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_n) \right)
\]

Setting the interest rate leaning against inflation and inflationary tensions stabilises inflation at the target of the central bank.

Going back to our simplified, 2 periods model, deviation of prices from the price target \(p^p\) will be smaller, the larger \(\phi_p\), in the reaction function of the central bank.

\[
y = \bar{y}_n - \sigma \left[ i - (p^p - p) \right] + \sigma \ln \beta + \text{shock} \quad \text{(Benigno 21)}
\]
\[
p - p^p = \kappa(y - y_n) \quad \text{with} \quad \kappa = \frac{1 - \alpha}{\alpha} (\eta + 1/\sigma) \quad \text{(Benigno 20)}
\]
\[
i = \bar{i} + \phi_p(p - p^p)
\]
\[
p - p^{p^p} = \kappa(\bar{y}_n - \sigma \left[ i - (p^{p^p} - p) \right] + \sigma \ln \beta + \text{shock} - y_n)
\]
\[
p - p^{p^p} = \kappa(\bar{y}_n - \sigma \left[ \bar{i} + \phi_p(p - p^{p^p}) - (p^{p^p} - p) \right] + \sigma \ln \beta + \text{shock} - y_n)
\]
\[
p - p^{p^p} = \frac{1}{1 + \kappa \sigma \phi_p - 1} \kappa(\bar{y}_n - \sigma \left[ \bar{i} + \sigma \ln \beta - y_n + \text{shock} \right]
\]

2.3 The inflation/output stabilization trade off

What should central banks do / business cycle and inflation fluctuations?

Based on Rudebush (2001) "Is the Fed too Timid?"

How to set the coefficients of the Taylor rule?

\[
i_t = \rho i_{t-1} + (1 - \rho) \left( \bar{r} + \pi^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_n) \right)
\]

How does this relate to the preferences of the central bank:

\[
Loss_{CB} = Var(\pi_t - \pi^*) + \lambda Var(y_t - y_n) + \nu Var(i_t - i_{t-1})
\]
\( \lambda \) and \( \nu \) reflect the preferences of the central bank.

Rudebush makes an interesting empirical exercise: how has the fed conducted interest rates (i.e. what has been the values of \((\rho, \phi_x, \phi_y)\), as obtained by OLS estimation of the Taylor rule, and how does it compares to the values one would obtain by minimising \( \text{Loss}_{CB} \).

To derive the optimal values of the \((\rho, \phi_x, \phi_y)\), one needs an evaluation of the transmission mechanism, i.e. the effects of changes in the nominal interest rate on inflation and the output gap.

Effectively, Rudebush estimates a "VAR" of inflation, output and the interest rate:

\[
\begin{align*}
\pi_t &= 0.2 + 0.9\pi_{t-1} + 0.05y_{t-1} + \varepsilon_t \\
y_t &= 0.2 + 0.8y_{t-1} - 0.1(i_{t-1} - \pi_{t-1}) + \eta_t \\
\varepsilon_t &\sim N(0, 1.2) \\
\varepsilon_t &\sim N(0, 0.9)
\end{align*}
\]

Note that the model is backward looking. Discuss this simplification.

For next week, compute the values of \( \text{Loss}_{CB} \) for three sets of preferences and three sets of coefficients of the central bank reaction function:

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \nu )</th>
<th>( \rho, \phi_x, \phi_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1, 1.5, 0.5</td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>0.9, 1.5, 0.5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.9, 1.5, 3</td>
</tr>
</tbody>
</table>

To get the value of \( \text{Loss}_{CB} \) simulate the model for 1000 periods to compute the variances of the variables of interest.

Compare to the conclusions of Rudebush.

2.3.1 Ex post: striking empirical evidence on the stabilization of inflation

See

- Diron and Mojon, www.benoitmojon.com/research
- Focus on the euro area inflation expectations

Key take away is the role of expectations for inflation (and other macroeconomic variables).

The "nice years" (Benati and Goodhart) of great moderation and the build up of financial imbalances.

However:

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• Friedman, long variables lags
• Orphanides on the diagnostic in real time
• many others in a vast, never ending literature,...
• totally out of touch with the post Lehman crisis, its build up; first order failure
3 Lecture: The transmission mechanism of monetary policy

3.0.2 Outline

1. Introduction
2. The yield curve
3. Interest rates and asset prices
4. The credit channel
5. Measuring the effects of monetary policy

3.1 Introduction

- Recall the AD curve in Benigno.
  How do changes in interest rate impact demand and how large is this impact? Purpose of today’s lecture.

- Money supply and the level of interest rates Central banks decide on, price of liquidity, i.e. the short-term interest rates on the interbank market.
  For instance, the ECB is auctioning liquidity to banks twice a week.

- More on liquidity in lecture 5

- Overview of the transmission mechanism

3.2 The yield curve

3.2.1 The expectation hypothesis

The short-term rates are entry point of the yield curve where, through arbitrage, longer maturity interest rate are tied to expected short-term interest rate, e.g. the 3 month interest rate

\[ i_{3m,t} \approx \frac{1}{3} (i_{1m,t} + E_t [i_{1m,t+1}] + E_t [i_{1m,t+2}]) + \xi_{3m,t} \]

Likewise, longer maturity interest rates, e.g. 10 years,

\[ i_{120m,t} \approx \frac{1}{120} E_t \left[ \sum_{k=1}^{120} i_{1m,t+k} \right] + \xi_{120m,t} \]

\( \xi_{3m,t} \) and \( \xi_{120m,t} \) are deviation from the pure expectation approach to long term interest rate for risk neutral investors. This will be (has been) covered in asset pricing classes. Not here.
Figure 14: Monetary policy transmission
The main take away point, however, is that future short-term interest rates will be set by the central bank as the price of central bank money (liquidity, reserves) for interbank transactions. This interest rate is the ultimate benchmark for the risk free interest rate.

Beyond the expectation hypothesis

- If the Expectation hypothesis would hold, the slope of the yield curve would tell us about future short term interest rates!!

- Effectively the long term rates are "on average" superior to short term rates and it is not the case that short term rates keep on rising: hence, there must be a part of the long term interest rate that is not the expectation of future short term interest rates:

\[
i_{120m,t} \approx \frac{1}{120} E_t \left[ \sum_{k=1}^{120} i_{1m,t+k} \right] + \xi_{120m,t}
\]

\[
\text{Mean}(\xi_{120m,t}) > 0
\]
• **Exercise**: compare the mean of interest rates at various maturities for DE, FR, IT, EA, before 2007, since then...

• $\xi_{120m,t}$ is a risk premia. Effectively, the longer the maturity the larger the fluctuation in the value of a bond in response to interest rates.

• If the risk premia were constant, changes in the slope of the yield curve (typically the difference between the 10 year and the 3 month interest rate) would predict future changes in the short term rate. In fact, a vast empirical literature has shown there is no such predictive ability. Hence, the risk premia is varying over time.

• Easy cases of changes in risk premia: credit risk over the business cycle. In the current (2011-2013) situation in the euro area: negative risk premia for Swiss, German and French bonds echo

• Try to think of situation where changes in the risk premia could for a credit risk free interest rate. What type of macroeconomic scenario could generate it?

• For next week, compute the average risk premia and its standard deviation in a 10 year euro area interest rate / EONIA. (data are in the class database)

3.2.2 Models of the yield curve

**Factor models of the yield curve** Basic intuition: extract the comovements of the yield curve through principal components analysis.

Exercise for next week:

• Extract the first four principal components of US and French interest rates of maturity 1, 3, 6, 12, 24, 60 and 120 months

• What is the share of the variance of rates they explain

• Graph the first 3 PC together with interest rates

**Nelson-Siegel pricing formulae** **Definition**: let the zero coupon yield at horizon $ZR_t^h$ be defined as

$$P(t, h) = \left( \sum_{i=1}^{h} \frac{C_i}{(1 + ZR_t^i)^i} \right) + \frac{100}{(1 + ZR_t^h)^{h}}$$

where is the price of the bond that pays in principal at horizon $h$ and, the sequence of coupons at each period. A zero coupon bond pays no coupon and only the principle (and de facto interest flows) upon its final reimbursement, i.e. the interest rate is defined through the price of the bond. French OAT are ZC bonds.

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Nelson and Siegel introduce a generic formulae of the yield curve as a function of the horizon of the bond and 3 parameters:

\[ ZR_{NS,t}^h = \beta_{0,t} + \beta_{1,t} \left[ 1 - \exp \left( \frac{-h}{\tau_1} \right) \right] \left( \frac{-h}{\tau_1} \right)^{-1} \]

\[ + \beta_{2,t} \left[ 1 - \exp \left( \frac{-h}{\tau_1} \right) \right] \left( \frac{-h}{\tau_1} \right)^{-1} - \exp \left( \frac{-h}{\tau_1} \right) \]

The parameters of the model are typically estimated minimizing the distance of the model to realized yield curve across maturities, for instance:

\[ \beta_{0,t} = \arg \min_{\beta_{0,t}, \beta_{1,t}, \beta_{2,t}} \sum_{i=1}^{N} (ZR_i^t - ZR_{NS,t}^h)^2 \]

In principle you can do it for any \( t \) if you have enough maturities or conduct a Kalman filter to generate the evolution of \( \beta_{0,t}, \beta_{1,t}, \beta_{2,t} \) as the unobserve state.

At the end of the day, applying the formulae give you the possibility to price the bond at any maturity.

### 3.3 Money and other financial instruments / asset prices

#### 3.3.1 The exchange rate channel

The central bank interest rate is also the marginal cost of expending the balance sheet of banks. It is therefore the benchmark for the cost of credit.

It is also the key in the pricing of assets:

The exchange rate, see the uncovered interest rate parity condition

\[ \Delta e_t = i_t - i_t^* + \varepsilon_t \]

#### 3.3.2 Stock prices

Stock prices are the discounted sum of future dividends

\[ s_t = E_t \left[ \sum_{k=1}^{end} \frac{d_{t+k}}{1 + i_t} \right] \]

#### 3.3.3 The wealth effect of monetary policy

Discuss
3.4 The credit channel

Bernanke and Blinder (1995) on the intensive and the extensive margin:
"that 5 percent of the population stop buying durables or investing, because they become credit constrained, can be more relevant for the business cycle than the [euler equation type of] marginal change in expenditure due to a marginal increase in interest rates". Go back to the Euler equation that founds the AD equation in Benigno.

3.4.1 The narrow credit channel/ the bank lending channel

- Kashyap and Stein series of papers on the US: Two steps to the analysis, some banks are cut off from financing and some borrowers are bank dependant.
  Kashyap and Stein AER-2000: A case of using cross section for identification
- Ehrman et al (2003): Evidence for the euro area prior to the crisis
- Gilchrist and Mojon (2012): Evidence for the euro area in times of crisis

3.4.2 The broad credit channel

- Use Bernanke (2003) in Angeloni et al. toy model: the secondary market price for collateral drops in time of recession
- Certainly relevant in times of recession; yet what is the effect of monetary policy itself? Evidence from Mojon, Smets and Vermeulen

3.5 Measuring the effects of monetary policy

- How to define a change in the stance of monetary policy
- A quasi natural experiment: the case of France 1723 (Velde, JPE 2009, Chronicle of an deflation unforetold)

3.5.1 Endogeneity and the VARs solution

The "exogenous part of the interest rate" is the part that cannot be seen as the endogenous/regular/systematic response of the policy instrument to the economic environnement

\[ i_t = \rho i_{t-1} + (1 - \rho) \left( \bar{r} + \pi^* + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t - y_n) \right) + \varepsilon_t^{MP} \]

One way, actually the most widely used way to evaluate the effects of monetary policy is to use a VAR model in which you have the objective variables of interest and the policy instrument.
Let's consider a simple NK inspired 3 variable economy VAR of order 2
\[
\begin{pmatrix}
    \frac{dy_t}{dt} \\
    \frac{dp_t}{dt} \\
    i_t
\end{pmatrix}
= \begin{pmatrix}
    a & b & c \\
    d & f & g \\
    h & j & k
\end{pmatrix}
\begin{pmatrix}
    \frac{dy_{t-1}}{dt} \\
    \frac{dp_{t-1}}{dt} \\
    i_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
    k' & l & m \\
    n & o & p \\
    q & s & w
\end{pmatrix}
\begin{pmatrix}
    \frac{dy_{t-2}}{dt} \\
    \frac{dp_{t-2}}{dt} \\
    i_{t-2}
\end{pmatrix}
+ \begin{pmatrix}
    \varepsilon_{1,t} \\
    \varepsilon_{2,t} \\
    \varepsilon_{3,t}
\end{pmatrix}
\]

in matrix form we have

\[
Y_t = A_1 Y_t + A_2 Y_t + \varepsilon_t
\]

we can actually rewrite this VAR of order 2 as a VAR of order 1 as follows:

\[
Z_t = \begin{pmatrix}
    Y_t \\
    Y_{t-1}
\end{pmatrix}
= \begin{pmatrix}
    \frac{dy_t}{dt} \\
    \frac{dp_t}{dt} \\
    \frac{dt}{dt} \\
    \frac{dy_{t-1}}{dt} \\
    \frac{dp_{t-1}}{dt} \\
    \frac{dt}{dt} \\
    \frac{dy_{t-1}}{dt} \\
    \frac{dp_{t-1}}{dt} \\
    \frac{dt}{dt}
\end{pmatrix}
\]

and \( \zeta_t = \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix} \)

\[
Z_t = B Z_{t-1} + \zeta_t
\]

with

\[
B = \begin{pmatrix}
    a & b & c & k' & l & m \\
    d & f & g & n & o & p \\
    h & j & k & q & s & w \\
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

**Proposition 1** Reminder of properties of the simplest AR(1) process and the lag operator:

\[
y_t = \phi y_{t-1} + \nu_t = \phi L y_t + \nu_t
\]

\[
y_t(1-\phi L) = \nu_t
\]

\[
y_t = (1-\phi L)^{-1} \nu_t
\]

for \( \phi < 1 \), this polynomial of the lag operator is invertible, hence

\[
y_t = \sum_{j=0}^{\infty} (\phi L)^j \nu_t = \nu_t + \phi L \nu_t + \phi^2 L^2 \nu_t + \ldots + \phi^n L^n \nu_t + \ldots
\]

\[
= \nu_t + \phi \nu_{t-1} + \phi^2 \nu_{t-2} + \ldots + \phi^n \nu_{t-n} + \ldots
\]

The smaller \( \phi \), the faster the decay, the faster the effects of a given shock \( \nu_t \) die out.
This inversion of the simplest AR process can be transposed in the context of the VAR, treating the matrix $B$ of the cannonical form of the VAR as $\phi$. The condition of invertibility will depend on the eigen values of the $B$ matrix. The largest "root/eigen value" of the $B$ matrix must have a modulus inferior to 1.

We can now more easily invert the "AR" form of the cannonic form of the VAR:

$$Z_t = BZ_{t-1} + \zeta_t$$
$$Z_t = (I - BL)^{-1}\zeta_t$$
$$Z_t = \sum_{j=0}^{\infty} C_j \zeta_{t-j} = \sum_{j=0}^{\infty} B^j \zeta_{t-j}$$

Hence, the realisations of the variables can be represented as the dynamic effects of past realisation of the past innovations $\varepsilon_{t-k}$. This will be directly used to represent estimates of the impulse response functions of a given variable (say inflation or the interest rate) to an identified shock, such as a monetary policy shock, a "demand" or a "supply" shock.

The IRFs of variable $m$ to shock $k$ be given by the sequences of a element $m,k$ of the $C_j$ matrices:

$$c_{m,k,j} \text{ for } j = 0, ..., h$$

Likewise, provided that we can transform the innovations $\varepsilon_t$ into economically meaningfull "shocks" $u_t$ that we construct as orthogonal to one another, we can use the MA representation to decompose the variance of economic variables the various economic shocks that we have included in the model.

$$Z_t = \sum_{j=0}^{\infty} C_j P \zeta_{t-j}$$
$$u_t = P \zeta_t \sim N(0, D), \text{ with } D \text{ a diagonal matrix}$$

Illustrate with evidence from Christiano, Eichenbaum and Evans, JPE 2004.

- For next week, estimate a VAR of euro area variables and describe the effects of "exogenous deviations of the interest rate", i.e. of a monetary policy shocks.
4 Lecture: The zero lower bound / liquidity trap

Outline

1. Introduction: a phenomena that plagues the EA, the US, Japan and the UK
2. Representing the ZLB in Benigno's model
3. Monetary policy responses
4. The fiscal multiplier at the ZLB

4.1 Introduction

- The japanese precedent
- The US household, the UK households and the Spanish private sector debt overhang => debt overhang and "deleveraging" weigh on demand
- According to some estimates, we would need very negative real interest rates to equate demand and supply. With inflation stuck at 2 %, the minimal interest rate we can reach is -2%.
- Fear of a Fisherian debt-deflation spiral (not what we saw in Japan, nor in the US or the Euro area.)
- Reaction of central banks/ monetary policies?
- Potency of fiscal policy?

4.2 The ZLB in the Benigno model

4.3 Unconventional monetary policy responses to the ZLB

Once you have exhausted the traditional instrument of monetary policy, when the nominal interest rate is at zero, what did central banks do? What are they doing currently.

Effectively, lower the level of other "important" interest rates.

In the NK model, demand depends entirely on the future path of short-term rate, i.e. on the long term interest rate:

\[
\begin{align*}
y_t &= \beta E_t [y_{t+1}] - \sigma (i_t - E_t [\pi_{t+1}]) \\
y_{t+1} &= \beta E_t [y_{t+2}] - \sigma (i_{t+1} - E_t [\pi_{t+2}]) \\
y_t &= \beta^2 E_t [y_{t+2}] - \sigma (i_t - E_t [\pi_{t+1}]) - \beta \sigma E_t (i_{t+1} - E_t [\pi_{t+2}]) \\
y_t &= \beta^n E_t [y_{t+n}] - \sigma \sum_{k=0}^{n} \beta^k E_t [i_{t+k} - \pi_{t+k+1}] 
\end{align*}
\]
In the real world, various interest rates matter and not only the short-term risk free interest rate.

Increasing the "debt of the central bank", implies an increase in the size of its balance sheet.

4.3.1 The euro area 6 main NC MP measures

Bank dominated economy. 85% of Non Financial Corporation debt consists of bank loans (against 33% in the US). Most measures are hence targeted on banks

1. FRFA - October 2008, right after Lehman, central bank money is auctioned to banks at fixed rate full allotment tenders. Not reversed as of February 2013.

2. VLTRO 1 - In May 2009, announced 12 months maturity auctions for June, September and December 2010. At least 2 effects: a) lower the cost of bank funding, including through reducing uncertainty b) pull down the interest rate for 18 month in May 2009.

3. SMP 1 in May 2010: buy Greek, Portuguese and Irish public debt; and SMP 2 in August 2011: buy Italian and Spanish debt

4. 36 months VLTRO in December 2011

5. OMT in September 2012: commitment to buy public debt of 1 to 3 years of maturity of countries who will seek the support of the European Stability Fund (i.e. a euro area IMF)

6. Forward guidance since July 2013: "keep rates at current levels of below for an extended period of time"

The US Forward guidance + intervention directly by purchasing assets on financial markets

- Credit easing
  In 2009, purchase of "agency bonds" and corporate paper;

- Quantitative easing/ Large asset purchase
  1. 2009, 600 billions of MBS and treasuries
  2. 2011, Operation twist
  3. 2012: 85 billions purchase of MBS and treasuries per month; "tapering", reduce monthly purchase by 10 billions starting in January 2014

- Forward guidance of future rates
  another case of inflation expectations
1. extended period of time
2. August 2011: fixed term, no rate increase until 2013
3. December 2012: until unemployment is inferior to 6.5 % and forecasted inflation below 2%
4. December 2013: well beyond unemployment is below 6.5% as long as forecasted inflation remains below 2%

4.3.2 The case for price level targeting (and nominal GDP targeting)
see Charles Evans (Chicago Fed president) speech in Boston, 16 October 2010.

4.4 The fiscal multiplier at the ZLB
Main conclusion: the fiscal multiplier would be much larger when the economy is at the Zero Lower Bound.
Getting the intuition in the Benigno AD, AS, monetary policy framework:

The flexible prices classical benchmark

\[ C_t + G_t = Y_t \]
\[ Y_t = F(L_t) = A_t L_t \]

The first order conditions yield:

\[ \frac{v'(L_t)}{u'(C_t)} = \frac{W}{P} \]
\[ F'(L_t) = \frac{W}{P} \]

Hence

\[ \frac{v'(L_t)}{u'(C_t)} = F'(L_t) = A \]

and

\[ \frac{v'(Y_t/A)}{u'(Y_t - G_t)} = A \]

with the standard functional form \( u(C_t) = C^{1-1/\sigma} / 1 - 1/\sigma \) and \( v(L_t) = L_t^{1+\eta}/1 + \eta \):

\[ (Y_t/A)^{\eta} = A(Y_t - G_t)^{-1/\sigma} \]

or taking small case as the logs of upper case ones
\[
\eta y_t + 1/\sigma y_t = 1/\sigma g_t + a(1 + \eta) \\
y_t = M_{Class} g_t + "bunch" \\
M_{Class} = \frac{1}{1 + \sigma \eta}
\]

Adding nominal rigidities (back to Benigno's model)
\[
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} - (\bar{p} - \check{p}) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta \quad \text{(Benigno 21)}
\]
\[
p - \check{p}^\ell = \kappa(y - y_n) \text{ with } \kappa = \frac{1 - \alpha}{\alpha} (\eta + 1/\sigma) \quad \text{(Benigno 20)}
\]

Substituting \( p \) into the AD equation gets the equilibrium level of output
\[
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} - (\bar{p} - \kappa(y - y_n) - \check{p}^\ell) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta \\
y = \frac{1}{1 + \sigma \kappa} \left[ \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} - (\bar{p} + \kappa y_n) - \check{p}^\ell) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta \right]
\]

Adding monetary policy

- **Accomodating monetary policy**

  Assume that we have a monetary policy which is fully accomodating fiscal policy. It adjusts the nominal interest rate in order to keep the real interest rate constant: it avoids the crowding out of additional demand. Then the multiplier is equal to 1.

  \[
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} - (\bar{p} - \check{p}) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta \quad \text{(Benigno 21)}
\]

- **Active monetary policy**

  Assume now that we have instead an active "lean against" inflation and output gap

  \[
i = \bar{i} + \phi_p (p - \check{p}^\ell) + \phi_y (y - y_c)
\]

  \[
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} + \phi_p (p - \check{p}^\ell) + \phi_y (y - y_c) - (\bar{p} - \check{p}) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta \\
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} + (\phi_p + 1)p - \phi_p \check{p}^\ell - \bar{p} + \phi_y (y - y_c) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta
\]

  and substitute again for the price level given by the AS equation

  \[
y = \tilde{y}_n + (g - \bar{g}) - \sigma \left[ \bar{i} + (\phi_p + 1) \left[ \kappa(y - y_n) + \check{p}^\ell \right] - \phi_p \check{p}^\ell - \bar{p} + \phi_y (y - y_c) - (\bar{\tau}_c - \tau_c) \right] + \sigma \ln \beta
\]
\[ y = \text{MNK} \left[ \frac{(g - \bar{g}) + (1 + \sigma \kappa (\phi_p + 1))\bar{g}_n}{-\sigma (\bar{y} + p^f - \phi_y \bar{y}_e - (\bar{\tau}_c - \tau_c))} \right] + \sigma \ln \beta \]

\[ M_{\text{NK}} = \frac{1}{1 + \sigma \kappa (\phi_p + 1) + \sigma \phi_y} \]

- **The case of a ZLB**

What happens in case we are hitting the zero lower bound on the nominal interest rate?

\[ i = 0 \]

Then

\[ y = \bar{y}_n + (g - \bar{g}) - \sigma [0 - (\bar{p} - \kappa (y - y_n) - p^f) - (\bar{\tau}_c - \tau_c)] + \sigma \ln \beta \]

\[ y = \frac{1}{1 + \sigma \kappa} \left[ \bar{y}_n + (g - \bar{g}) - \sigma [(\bar{p} + \kappa y_n) - p^f) - (\bar{\tau}_c - \tau_c)] + \sigma \ln \beta \right] \]

\[ M_{\text{ZLB}} = \frac{1}{1 + \sigma \kappa} \]

Now consider on top that the shock that takes depresses (negative output gap and deflation) the economy is persistent and that a fiscal policy response would also be persistent. In particular, fiscal policy induces an increase in future prices through its positive effect on future output gaps, i.e. it reduces the extend of deflation

\[ y = \bar{y}_n + (g - \bar{g}) - \sigma [0 - (\bar{p} - \kappa (y - y_n) - p^f) - (\bar{\tau}_c - \tau_c)] + \sigma \ln \beta \] (Beligno 21)

\[ y = \bar{y}_n + (g - \bar{g}) - \sigma [0 - \kappa g - (\bar{\tau}_c - \tau_c)] + \sigma \ln \beta \] (1)

\[ y = (1 + \sigma \kappa)g + \text{bunch} \] (2)

\[ M_{\text{ZLB}} = (1 + \sigma \kappa) \] (3)

- **Back of the envelop comparisons of multipliers**

Let us now focus on this multiplier for plausible values of the preference and "structural" parameters of the economy:

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\sigma)</th>
<th>(\eta)</th>
<th>(\kappa)</th>
<th>(\phi_p)</th>
<th>(\phi_y)</th>
<th>(M_{\text{classical}})</th>
<th>(M_{\text{NK}})</th>
<th>(M_{\text{ZLB}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>0.5</td>
<td>0.2</td>
<td>1.13</td>
<td>1.5</td>
<td>0.5</td>
<td>0.9</td>
<td>0.375</td>
<td>1.6</td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>0.73</td>
<td></td>
<td>0.9</td>
<td>0.46</td>
<td>1.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.36</td>
<td>0.66</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.54</td>
<td>0.5</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.66</td>
<td>0.5</td>
<td>0.2</td>
<td>1.13</td>
<td>0.8</td>
<td></td>
<td>0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.13</td>
<td>1</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.5</td>
<td>0.2</td>
<td>0.73</td>
<td>0.8</td>
<td>0.5</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.4.1 The "scandal" of the October 2012 IMF World Economic Outlook

Discussion.

Preview: the US debate on the Fiscal stimulus in 2009 (Christina Romer, adviser of Obama)

In the context of the Greek/Spanish and Italian fiscal consolidation packages, the larger the fiscal multiplier the more difficult and risky severe fiscal adjustment programs. See also Blanchard and Leigh early 2013 IMF working paper.
5 Lecture: Financial intermediation and central banks balance sheet

5.0.2 Outline

Introduction

1. Money and financial intermediation
2. A bank balance sheet and profit and loss account approach: Monti-Klein
3. An (asymmetric) information approach: Holmstrom-Tirole
4. Ongoing substitutions between private and public monies: the balance sheet policies of the ECB and the Fed

5.1 Introduction: What is money?

- Monetary policy is the supply of money
  
  Money is a debt that is used to settle other debts. It is the legal tender for transactions.
  
  The transaction role of money and the double coincidence of needs. (Complementary reading: *Money is memory, Kocherlakota*).
  
  Money is issued by central banks who were initially the banker of the sovereign who granted them the privilege of issuing money in exchange of favorable financing conditions.
  
  During the 19th century, they became the bank of banks. Payments systems are hierarchical.
  
  Financial crises and the Lender of Last Resort. (Complementary reading: *Central Banking, Aglietta and Mojon*)

5.2 A balance sheet view of financial intermediation

5.2.1 Financial intermediation
### Financial Intermediation

<table>
<thead>
<tr>
<th>Bank A</th>
<th>Bank B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
</tr>
<tr>
<td>Reserves 9</td>
<td>Deposits (of H) 90</td>
</tr>
<tr>
<td>mortgage 50</td>
<td></td>
</tr>
<tr>
<td>CD 41</td>
<td></td>
</tr>
<tr>
<td><strong>Capital</strong> 10</td>
<td></td>
</tr>
<tr>
<td><strong>Household H, client of A</strong></td>
<td><strong>NFC F, client of B</strong></td>
</tr>
<tr>
<td>assets</td>
<td>liabilities</td>
</tr>
<tr>
<td>House 60</td>
<td>mortgage 50</td>
</tr>
<tr>
<td>Stocks 40</td>
<td></td>
</tr>
<tr>
<td>Deposits 90</td>
<td></td>
</tr>
<tr>
<td><strong>Net wealth</strong> 140</td>
<td></td>
</tr>
</tbody>
</table>

### Banking System

<table>
<thead>
<tr>
<th>assets</th>
<th>liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves 10</td>
<td>Deposits 100</td>
</tr>
<tr>
<td>Loans + mortgage 90</td>
<td></td>
</tr>
<tr>
<td>CD 41</td>
<td>CD 41</td>
</tr>
<tr>
<td><strong>Capital</strong> 20</td>
<td></td>
</tr>
<tr>
<td><strong>credit</strong> 110</td>
<td><strong>money</strong> 100</td>
</tr>
</tbody>
</table>

### Central Bank

<table>
<thead>
<tr>
<th>assets</th>
<th>liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves 10</td>
<td>Reserves 10</td>
</tr>
<tr>
<td>Gold + forex 10</td>
<td>notes 10</td>
</tr>
</tbody>
</table>
### Increase in credit and hence money

<table>
<thead>
<tr>
<th>Bank A</th>
<th>Bank B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
</tr>
<tr>
<td>Reserves</td>
<td>Deposits (of H)</td>
</tr>
<tr>
<td>mortgage</td>
<td>Foreign borrow</td>
</tr>
<tr>
<td>CD</td>
<td>72</td>
</tr>
<tr>
<td>Capital</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Household H, client of A</th>
<th>NFC F, client of B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
</tr>
<tr>
<td>House</td>
<td>60</td>
</tr>
<tr>
<td>Stocks</td>
<td>40</td>
</tr>
<tr>
<td><strong>Deposits</strong></td>
<td><strong>120</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Net wealth</strong></td>
<td><strong>170</strong></td>
</tr>
</tbody>
</table>

**5.2.2 Monetary/credit creation**

**Discuss**

1. Credit makes deposits?
2. How do reserve requirements limit monetary creation?
3. Two instruments of monetary policy: changes in reserve requirements and the interest rate on reserves: the supply of reserves?
## Banking system

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>14</td>
</tr>
<tr>
<td>Loans + mortgage</td>
<td>130</td>
</tr>
<tr>
<td>CD</td>
<td>72</td>
</tr>
<tr>
<td>Bonds</td>
<td>20</td>
</tr>
</tbody>
</table>

### Credit: 150

### Money: 140

## Central bank

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>14</td>
</tr>
<tr>
<td>Gold + forex</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>14</td>
</tr>
<tr>
<td>Notes</td>
<td>10</td>
</tr>
</tbody>
</table>
5.3 The Monti-Klein model of banking: an IO perspective

The balance sheet of the bank is given by

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves $R$</td>
<td>Deposits $D$</td>
</tr>
<tr>
<td>Loans $L$</td>
<td>Capital $K$</td>
</tr>
</tbody>
</table>

The reserves are typically (traditionally) paid no interest. It is a "tax" on financial intermediation and banks try to keep it as low as possible:

$$R \geq \alpha D$$

$$R = \alpha D$$

This model assumes a monopolistic bank which is collecting deposits, confronted to an upward slopping deposit supply $D^S = D(r_D)$ or its inverse function $r_D(D)$, and supplying loans, confronted to a downward slopping loan demand $L^D = L(r_L)$ or its inverse function $r_L(L)$:

Let us now consider the profit function of this monopolistic bank assuming its capital as given and the level of the money market rate $r$ also as given:

$$\Pi(L, D) = (r_L(L) - r)L + (r(1 - \alpha) - r_D(D))D - C(D, L)$$

$C(D, L)$ is a cost function for doing financial intermediation, essentially about the non financial (independent from interest rates) costs such as network of branches, advertisement, recruiting, IT, ...

The first order conditions are

$$\frac{\partial \Pi}{\partial L} = r'_L(L)L + (r_L(L) - r) - C'_L(D, L) = 0$$

$$\frac{\partial \Pi}{\partial D} = -r'_D(D)D + r(1 - \alpha) - r_D(D) - C'_D(D, L) = 0$$

Introducing the elasticities of loan demand and deposit supply:

$$\varepsilon_L = \frac{r_LL'(r_L)}{L(r_L)}; \varepsilon_D = \frac{r_DD'(r_D)}{D(r_D)}$$

The profit maximizing interest rate margins of the banks are

$$\frac{r'_L - (r + C'_L)}{r'_L} = \frac{1}{\varepsilon_L(r'_L)}$$

$$\frac{r(1 - \alpha) - r'_D - C'_D}{r'_D} = \frac{1}{\varepsilon_D(r'_D)}$$

Main results:

1. If $C(D, L)$ is additive, i.e. the cost of loan supply and deposit collection are "indepedent", the profit maximizing interest rate margins on loans and on deposits are independent from one another.
2. Changes in the money market interest rate pushes up both \( r^D_L \) and \( r^L_L \).

Hence, in this representation of bank’s behavior, we see that the transmission mechanism of monetary policy can be very simple as intermediated interest rates respond mechanically to the money market interest rate.

Main short-comings:

1. Ignores credit risks and information asymmetries. Stiglizt and Weiss (1981) apply contract theory/adverse selection to introduce the notion that higher interest rate may deteriorate the average quality of borrowers. At very high interest rates you risk attracting only borrowers who anticipate they are unlikely to repay you. The "interest rate" on loan may hence turn out not market clearing. (a point we will see in lecture 6). An interesting exercise, and potential exam question, would consist of introducing credit risks in the Monti Klein model.

2. Banks themselves may take too much risk. This is one motivation for capital requirements, liquidity requirements (in addition to required reserves) and other bank regulation/supervision. (a point we will see in lecture 7)

5.4 The Holmstrom-Tirole model of Banking (loanable funds- QJE 1997): an asymmetric information perspective

The Holmstrom-Tirole model is simple application of contract theory to define what banks do and why financial intermediation exists at all. This section reproduces the synthetic presentation of the model by Freixas and Rochet section 2.5.3.

5.4.1 The model set up

3 agents, firms who borrow to invest in risky projects; banks who collect deposits and grant loans to firms and uninformed investors.

Firms have projects that cost \( I \) and return \( R \). Good firm projects have high probability \( p_H \) of generating \( R \). bad firm project have low probability \( p_L \) of generating \( R \).

**Moral hazard**: firms can choose a bad project that gives a private benefit \( B \); however, this private benefit can be reduced to \( b \) in case banks can incur a cost \( C \) of auditing the firm.

Uniformed investors can earn a risk free alternative return of \( \gamma \).

We assume that only good projects have positive net value, i.e.

\[
p_H R - \gamma I > 0 > p_L R - \gamma I + B
\]

Finally, firms have more or less of their own capital \( A \) to invest in their projects.
5.4.2 Direct finance (no bank involved)

Uniformed investors are offered $R_u$ for an $I_u$ investment. But the residual return that accrue to the firm has to be large enough, the incentive compatibility constraint is given by

$$p_H(R - R_u) > p_L(R - R_u) + B$$

$$R_u \leq R - \frac{B}{\Delta p}, \Delta p = p_H - p_L$$

The individual participation constraint for investors implies

$$p_H R_u > \gamma I_u$$

$$I_u \leq \frac{p_H R_u}{\gamma} \leq \frac{p_H}{\gamma} \left[ R - \frac{B}{\Delta p} \right]$$

Hence, a project can be financed only if the entrepreneur can cover the $I - I_u$ gap.

$$A + I_u > I$$

$$A \geq \bar{A}(\gamma) = I - \frac{p_H}{\gamma} \left[ R - \frac{B}{\Delta p} \right]$$

5.4.3 Indirect finance, through a financial intermediary

The incentive compatibility constraint of the firm:

$$p_H(R - R_m - R_u) > p_L(R - R_m - R_u) + b$$

$$R_m + R_u \leq R - \frac{b}{\Delta p}$$

The incentive compatibility constraint of the bank:

$$p_H R_m - C \geq p_L R_m$$

$$R_m \geq \frac{C}{\Delta p}$$

Assuming $\beta$ is the reservation (alternative) return that the bank can have on financial markets, and firms always exhaust direct finance before turning to banks:
\[ R_m = \frac{C}{\Delta p} \]

\[ I_m(\beta) = \frac{p_H R_m}{\beta} = \frac{p_H C}{\beta \Delta p} \]

\[ R_m + R_u \leq R - \frac{b}{\Delta p} \]

\[ R_u \leq R - \frac{C + b}{\Delta p} \Rightarrow I_u \leq \frac{p_H}{\gamma} \left[ R - \frac{C + b}{\Delta p} \right] \]

A larger proportion of firms (with lower level of initial capital) could be in a position to see their project financed as:

\[ A + I_u + I_m > I \]

\[ A > \Delta(\beta, \gamma) = I - I_m(\beta) - \frac{p_H}{\gamma} \left[ R - \frac{C + b}{\Delta p} \right] \]

We hence have 3 categories of firms depending on their own level of capital \(A\):

1. \(A < \Delta(\beta, \gamma)\) no financing
2. \(\Delta(\beta, \gamma) < A < \tilde{A}(\gamma)\) : bank financing
3. \(\tilde{A}(\gamma) < A\) : direct finance.

5.4.4 Holmstrom-Tirole in the Busineses/Financial Cycle

Discuss in the context of this model

- The broad credit channel
- The "narrow" bank lending channel
- How buying assets can help the financing of the economy.
- Take the model to identify the nature of the forces that dominate the financial system:

1. A credit crunch (a negative bank lending supply shock) would decrease \(\gamma\) and increase \(\beta\);
2. A collateral squeeze, i.e. a drop in firms assets would decrease both interest rates;
3. A savings squeeze would increase \(\gamma\) anad decrease \(\beta\).
5.5 The Bernanke Blinder model of banking and macro

Remark: IS-LM plus credit (Bernanke and Blinder; AER 1988)

The model is an extension of the IS-LM framework to include in the model credit as a third asset/financial instrument in addition to money and government bonds.

Loan (i.e. bank credit) demand equation

\[ L^d = L(\rho, i, y) \]
\[ L^d_\rho < 0, L^d_y > 0, L^d_i > 0 \] (substitution)

The balance sheet of the banking sector, assets are either loans, excess reserves or bonds:

\[ L + E + B = (1 - \tau)D \]
\[ R = \tau D \]

The credit supply can take the form:

\[ L^s = \lambda(\rho, i)(1 - \tau)D \]
\[ \lambda^s_\rho > 0, \lambda^s_i < 0 \]

The equilibrium on the loan market implies

\[ L(\rho, i, y) = \lambda(\rho, i)(1 - \tau)D \]

Let the demand for excess reserve, which are not remunerated, be

\[ E = \varepsilon(i)(1 - \tau)D \]
\[ \varepsilon^i < 0 \]

the money multiplier is then

\[ m(i) = \frac{D}{R} = \frac{1}{\varepsilon(i)(1 - \tau) + \tau} \]

end the equilibrium for deposits is

\[ D(i, y) = m(i)R \]

Let the IS curve take the form

\[ y = Y(\rho, i) \]

Analysing the equilibrium on the credit market:
\[ L(\rho, i, y) = \lambda(\rho, i)(1 - \tau)D = \lambda(\rho, i)(1 - \tau)m(i)R \]
gives an equilibrium interest rate on loans \( \rho \)
\[
\rho = \phi(i, y, R)
\]
\[
\phi_i > 0, \phi_y > 0, \phi_R < 0
\]
intuitions:
• \( \phi_R' < 0 \) an increase in reserves, for a given money multiplier, increases loanable funds;
• \( \phi_y' > 0 \) an increase in the demand for loans pushes up their interest rate
• \( \phi_i' > 0 \), consider \( L_i' > 0, \lambda_i' < 0, \) and \( m_i' > 0 \); for an increase of loan demand (RHS) and a reduction of loan supply (LHS), and provided \( m_i' \) is small enough, the equation can hold only if \( \rho \) increases.

and the IS curve becomes
\[
y = Y(\phi(i, y, R), i)
\]

**Useful insights from the model** The purpose of this model is to have one more dimension / observable variable to analyse the dynamics of the economy and financial markets. Beyond the level of interest rates on government bonds, we can also observe interest rate differentials (i.e. spreads). Changes in these spreads are indications of the supply and demand for credit / bonds.

**Examples:**
• spreads on interbank markets, see the Euribor-OIS before and since August 2007
• Credit supply before the crisis
• Also, since the summer of 2011, the aversion to risk in financial markets is extreme such that Spain and Italy pay much higher interest rates to borrow than France and Germany. At 5 years horizon, Spain and Italy currently borrow at around 5% while France is above 2% and Germany bellow 2%. These are the lowest ever interest rate for these countries, reflecting a flight to quality.

5.6 Central banks balance sheet policy since Lehman

5.6.1 The balance sheet of the ECB in 2006 and 2012

The ECB has assumed an intermediation role that the interbank market had lost.
### Eurosystem balance sheet main items (30 November 2012, billions euros)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>Banknotes</td>
</tr>
<tr>
<td>Forex reserves non residents</td>
<td>Owed to euro area credit institutions</td>
</tr>
<tr>
<td>Forex reserves residents</td>
<td>Owed to other euro area resident</td>
</tr>
<tr>
<td><strong>Lending to EA credit institutions</strong></td>
<td><strong>Owed to non EA residents</strong></td>
</tr>
<tr>
<td>Main refinancing operation</td>
<td>75</td>
</tr>
<tr>
<td>Longer term RO</td>
<td>1040</td>
</tr>
<tr>
<td>Other claims in EA CI</td>
<td>revaluation account</td>
</tr>
<tr>
<td>Securities of EA residents</td>
<td>Capital and reserves</td>
</tr>
<tr>
<td><strong>inc for MP purpose</strong></td>
<td>277</td>
</tr>
<tr>
<td>Sub Total</td>
<td>3839</td>
</tr>
<tr>
<td>other stuff</td>
<td>Sub Total</td>
</tr>
<tr>
<td>Total</td>
<td>3033</td>
</tr>
<tr>
<td><strong>nominal GDP = about 9450 billions</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Eurosystem balance sheet main items (1 December 2006)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>Banknotes</td>
</tr>
<tr>
<td>Forex reserves non residents</td>
<td>Owed to euro area credit institutions</td>
</tr>
<tr>
<td>Forex reserves residents</td>
<td>Owed to other euro area resident</td>
</tr>
<tr>
<td><strong>Lending to EA credit institutions</strong></td>
<td><strong>Owed to non EA residents</strong></td>
</tr>
<tr>
<td>Main refinancing operation</td>
<td>307</td>
</tr>
<tr>
<td>Longer term RO</td>
<td>120</td>
</tr>
<tr>
<td>Other claims in EA CI</td>
<td>revaluation account</td>
</tr>
<tr>
<td>Securities of EA residents</td>
<td>Capital and reserves</td>
</tr>
<tr>
<td><strong>inc for MP purpose</strong></td>
<td>na</td>
</tr>
<tr>
<td>Sub Total</td>
<td>861</td>
</tr>
<tr>
<td>other stuff</td>
<td>Sub Total</td>
</tr>
<tr>
<td>Total</td>
<td>1126</td>
</tr>
</tbody>
</table>

5.6.2 Interpretations of the balance sheet policy (Keister-McAndrews)

The Fed (and the ECB) increase the supply of public liquidity when the provision/acceptability of private liquidity collapse.

(complementary readings: Holmstrom and Tirole’s Private and Public supply of liquidity; Brunnermeier and Sanikov’s The I theory of Money)
<table>
<thead>
<tr>
<th>Normal times</th>
<th>Bank A</th>
<th>Bank B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
<td><strong>assets</strong></td>
</tr>
<tr>
<td>Reserves</td>
<td>10</td>
<td>Deposits</td>
</tr>
<tr>
<td>Loans</td>
<td>50</td>
<td>Loans</td>
</tr>
<tr>
<td>Due from bank B</td>
<td>40</td>
<td>Deposits</td>
</tr>
<tr>
<td>Securities</td>
<td>10</td>
<td>Due to bank A</td>
</tr>
<tr>
<td>Capital</td>
<td>10</td>
<td>Capital</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Central bank intermediation type I, eg VLTROs</th>
<th>Bank A</th>
<th>Bank B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
<td><strong>assets</strong></td>
</tr>
<tr>
<td>Reserves</td>
<td>50</td>
<td>Deposits</td>
</tr>
<tr>
<td>Loans</td>
<td>50</td>
<td>Loans</td>
</tr>
<tr>
<td>Due from bank B</td>
<td>40</td>
<td>Deposits</td>
</tr>
<tr>
<td>Securities</td>
<td>10</td>
<td>Due to Eurosystem</td>
</tr>
<tr>
<td>Capital</td>
<td>10</td>
<td>Capital</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Central bank intermediation type II, QEs/LSAP/SMP/TA??</th>
<th>Bank A</th>
<th>Bank B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
<td><strong>assets</strong></td>
</tr>
<tr>
<td>Reserves</td>
<td>90</td>
<td>Deposits</td>
</tr>
<tr>
<td>Loans</td>
<td>50</td>
<td>Loans</td>
</tr>
<tr>
<td>Securities</td>
<td>10</td>
<td>Due to Eurosystem</td>
</tr>
<tr>
<td>Capital</td>
<td>10</td>
<td>Capital</td>
</tr>
</tbody>
</table>

The central bank acquires 40 of securities by issuing money.
6 Financial and credit cycles

6.0.3 Outline
1. The asset price/collateral/credit feedback loop
2. The maturity rat race
3. Minsky

6.1 The asset price/collateral/credit feedback loop

6.1.1 The Kiyotaki-Moore model
(Source: Freixas and Rochet 6.4.2)

Assumptions: 2 goods:
   a) a perishable good \( X \) that is either consummed or invested in production
   b) a capital good \( k \) such as real estate that can be used as an input into production and pledged as collateral

2 classes of agents
   i) entrepreneurs own the technology and the capital good \( k \)
   ii) lenders who receive an endowment of the consumption good

Entrepreneurs will borrow the consumption good they invest from lenders

Technology is Leontief
\[ X_{t+1} = \min(X_t, \lambda k_t) \]

Loans \( b_t \) must be fully collateralized, and this collateral constraint will be binding, with \( q_{t+1} \) the expected asset price:

\[
\begin{align*}
  b_t &\leq \frac{q_{t+1}k_t}{1 + r} \\
  b_t &= \frac{q_{t+1}k_t}{1 + r}
\end{align*}
\]

with \( r = 1/\beta - 1 \)

There are 2 demand for real estate, one is an input into production, seen above and the other one is rental for housing purpose.

With a fixed total supply of real estate the rental rate \( h_t \) is an increasing function of the real estate used for production \( A_t \):

\[ h_t = m(A_t + h_0) \]

Solving the model, what are \( A_t \) and \( q_t \) in equilibrium?

\[
A_t = \frac{\lambda A q_{t+1}}{1 + r}
\]

The no arbitrage condition for the price of land is:
\[ q_{t+1} + \frac{X - (1 + r) \lambda q_{t+1}}{1 + r} + h_t \left( 1 - \frac{\lambda q_{t+1}}{1 + r} \right) = (1 + r) q_t \]

substituting in \( A_t \) and \( h_t \)

\[ q_{t+1} + \frac{X - (1 + r) \lambda q_{t+1}}{1 + r} + m \left( \frac{\lambda A q_{t+1}}{1 + r} + h_0 \right) \left( 1 - \frac{\lambda q_{t+1}}{1 + r} \right) = (1 + r) q_t \]

basic algebra

\[
\begin{align*}
q_t &= a q_{t+1}^2 + b q_{t+1} + c \\
a &= -\frac{\lambda^2 m A}{(1 + r)^2} \\
b &= \frac{X - \lambda m (A - h_0)}{(1 + r)^2} \\
c &= \frac{m h_0}{1 + r}
\end{align*}
\]

We can describe a 2 period cycle in the plan \((q_{t+1}, q_t)\).

### 6.1.2 Relavance for the last crisis

Mian and Sufi: the aggregate demand

Midrigan and Philipon

### 6.2 The maturity rat race

Kashyap and Stein (2012): Optimal conduct of monetary policy with interest rate on reserves. This is a simpler version of Stein’s model where some of the key parameters are further microfounded.

#### 6.2.1 Motivation

One of the defining feature of the Lehman crisis is the maturity mismatch / liquidity risk.

Take the standard business model of CDOs, Northern rock, Bear Stearn

**Illustration of maturity mismatch**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBS</td>
<td>100 Capital</td>
</tr>
<tr>
<td>CDs</td>
<td>99</td>
</tr>
</tbody>
</table>

MBS typically have a 5 to 10 years maturity. CDs issued from overnight to 3 month. Like any financial intermediary, you the business model is to earn a higher yield on your asset than what you pay on your liabilities.

**Illustration of maturity mismatch compounded by credit risk**
6.2.2 The model

Three dates 0, 1 and 2

Time 0, each bank is endowed with fixed asset, its own capital
These assets pay a dividend at time 1 and a return at time 2
The dividend is uniformly distributed on \[0, K\]

Decision of the bank
How much to engage into maturity transformation activities: choosing the amount of an illiquid investment \(I\) that will pay \(\theta I\) at time 2

The bank finances a fraction \(m\) of the investment with short term maturity than need to be rolled over at period 1 and \((1 - m)\) with long term debt, for an \(R\). Instead, the roll over debt cost is only \(R - \Delta\).

The payoff of maturity transformation is

\[I(\theta - R + m\Delta)\]

A natural feature for the competitive equilibrium to exist is that

\[R - m\Delta < \theta < R\]

The form of liquidity risk With probability \(p\), bad state at period 1 and some banks won’t be able to service (reimburse) short-term debt.

Given that the dividends are uniform, the proportion of banks in distress in the bad state is \(mI/K\), a proxy of short term debt to capital.

Key assumption: for any bank, the deadweight costs of distress is increasing in the fraction of banks in distress (fire sale externality)

The fraction of bank in distress in the bad state will depend on \(\bar{I}\), the average of maturity transformation across all banks. We hence assume that for each bank, the deadweight cost is \(Z = \gamma \bar{I}\).

The net expected profit for each bank:

\[\Pi = I(\theta - R + m\Delta) - \frac{pm\gamma}{K}I\]

The externality An individual bank will not take into account the effect of its own decision on \(\bar{I}\). It will take \(\bar{I}\) as given.

FOC of the individual bank

\[I^* = (\theta - R + m\Delta)\frac{K}{pm\gamma}\]
In contrast, the social planner will internalise the effects of individuals on the fire sale externality.

FOC of the social planner

\[ I^{**} = (\theta - R + m\Delta) \frac{K}{2pm\gamma} \]

Policy: a pigouvian tax on maturity mismatch

\[ \Pi = I(\theta - R + m[\Delta - \tau]) - \frac{pmI\gamma I}{K} \]

Optimal private investment will get

\[ I^* = (\theta - R + m[\Delta - \tau]) \frac{K}{pm\gamma} \]

The optimal tax is

\[ \tau^{**} = \frac{\theta - R + m\Delta}{2m} \]

Discuss Regulation of liquidity in the Basel process

Recent paper by the NY Fed (Bai and Krisnamuthy)

6.3 A model of systemic bank crises / collapse of the interbank market

Boissay, Collard and Smets (2012)

6.3.1 Motivation

- Better understand the joint dynamics of regular business cycles and systemic banking crises (SBCs)
- Account for the few features common to SBCs (Reinhart and Rogoff, 2009; Jordà et al., 2011; Claessens et al., 2011; Schularick and Taylor, 2012) key stylised facts:
  1. SBCs are rare events – on average 1 every 40 years in OECD countries
  2. Recessions that follow SBCs are deeper and last longer – output loss during a SBC is 60% larger
  3. SBCs are "credit booms gone wrong"
  4. In the case of the subprime crisis, bad loans where not so big. But levered financial intermediaries were many.
Remark: In most DSGE models with financial frictions banking crises are big negative shocks amplified
Can explain Key Facts #1 & #2 but **Cannot explain Key Fact #3**
SBCs are not random
Explaining Key Fact #3 requires to model the economic dynamics leading to SBCs.

From a policy perspective, the framework is a step forward towards: DSGE–based crisis prevention policy analysis and DSGE–based early warning signals.

### 6.3.2 Stylized facts

SBCs are rare and bring about deep and long recessions

<table>
<thead>
<tr>
<th>Frequency, magnitude, and duration of systemic banking crises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (%)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>All banking crises</td>
</tr>
<tr>
<td>Systemic Banking Crises (SBC)</td>
</tr>
<tr>
<td>All recessions</td>
</tr>
<tr>
<td>Recessions with SBC (A)</td>
</tr>
<tr>
<td>Recessions w/o SBC (B)</td>
</tr>
<tr>
<td>Test A ≠ B, p-value (%)</td>
</tr>
</tbody>
</table>

Source: Schularik et al. (2011), data for 14 OECD countries, 1870-2008
Crisis defined as in Laeven and Valencia (2008)
6.3.3 The model

Representative Household and Firm

- Firm: \( \max_{\{k_t, h_t\}} \pi_t = F(k_t, h_t; z_t) + (1 - \delta)k_t - R_t k_t - w_t h_t \)
- Household:
  \[
  \max_{\{a_{t+1}, c_{t+1}, h_{t+1}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau u(c_{t+\tau}, h_{t+\tau})
  \]
  subject to budget constraint
  \[
  c_t + a_{t+1} = r_t a_t + w_t h_t + \pi_t
  \]

Notice that \( r_t \leq R_t \) (spread) and \( k_t \leq a_t \) (credit crunch)

The Banking Sector

- Banks are atomistic, competitive, and price takers
- Heterogeneous 1-period banks

- Bank \( p \)'s net return per unit of corporate loan is \( pR_t \)

Beneficial to relocate funds: unskilled banks lend to skillful banks on an interbank market. But relocation impaired due to:

Asymmetric information: \( p \) is private information
Moral hazard: bank \( p \) may borrow \( \phi_t \) and run away

- Bank \( p \) has 4 options:
  1. Lend to other banks on the interbank market \( \implies \rho_t \)
  2. Store goods \( \implies \gamma \)
  3. Raise funds \( \phi_t \) from market and lend to firm \( \implies pR_t (1 + \phi_t) - \rho_t \phi_t \)
  4. Raise funds \( \phi_t \) from market and walk away \( \implies \gamma (1 + \theta \phi_t) \)

- Notice that the incentive to divert depends on corporate loan \( R_t \)
  - The higher \( R_t \), the lower the incentive to divert
The Borrowing Bank’s Problem

- Borrowing bank \( p \) solves:
  \[
  \max_{\phi_t} r_t (p) \equiv p R_t (1 + \phi_t) - \rho_t \phi_t
  \]
  \[
  PC : \quad p R_t (1 + \phi_t) - \rho_t \phi_t \geq \rho_t \quad \Rightarrow p \geq \bar{p}_t \equiv \rho_t / R_t
  \]
  \[
  IC : \quad \gamma (1 + \theta \phi_t) \leq \rho_t \quad \Rightarrow \phi_t = (\rho_t - \gamma) / \theta \gamma
  \]

- Profits are fully distributed to household: \( r_t \equiv \int_0^1 r_t (p) \, d\mu (p) \)

Interbank Market Equilibrium

Interbank market clearing condition:

Supply \((+\)\)

\[
\mu (\bar{p}_t) = \frac{\left(1 - \mu (\bar{p}_t)\right)}{\phi_t}
\]

Demand bends backward \((+ or -)\)

"extensive margin" \((-\)\)

"intensive margin" \((+\)\)

with \( \bar{p}_t \equiv \rho_t / R_t \) and \( \phi_t = (\rho_t - \gamma) / \theta \gamma \)

Representation of the demand and supply on the interbank market

Return on equity and corporate loan supply

- Return on equity:
  \[
  r_t = \begin{cases} 
  R_t \int_{\bar{p}_t}^1 p \frac{dp}{\mu (\bar{p}_t)} , & \text{if an equilibrium with trade exists} \\
  R_t \left( \frac{\gamma}{R_t} \mu \left( \frac{\gamma}{R_t} \right) + \int_{\bar{p}_t}^1 p \, d\mu (p) \right) , & \text{otherwise.} 
  \end{cases}
  \]
Corporate loan supply

\[ k_t^* = \begin{cases} a_t, & \text{if an equilibrium with trade exists} \\ \left(1 - \mu \left(\frac{a_t}{\bar{a}}\right)\right) a_t, & \text{otherwise} \end{cases} \]

Absorption capacity and financial imbalances

**Proposition (Interbank loan market freeze):** The interbank loan market is at work if and only if \( a_t \leq \bar{a} \equiv f_k^{-1}(R_t + \delta - 1; z_t) \), and freezes otherwise.

- The interbank market improves efficiency but freezes when \( R_t < \bar{R} \)
- In general equilibrium, \( R_t \) is driven by savings \( (a_t) \) and technology \( (z_t) \). Hence the interbank market freezes when \( a_t > \bar{a}(z_t) \)
- **Threshold \( \bar{a}(z_t) \) is the banking sector’s "absorption capacity"**
- A measure of financial imbalances is \( \bar{a}(z_t) - a_t \)

Two-way relationship between the retail and the wholesale loan markets

- Whether the interbank market is functioning depends on the corporate loan market equilibrium rate \( R_t^* \)
- \( R_t^* \) depends on whether the interbank market is functioning
- The model must be solved taking these interactions into account:
  1. Conjecture the interbank market operates and solve for \( R_t^* \)
  2. Verify whether indeed the interbank market operates \( (R_t^* \geq \bar{R}) \)
  3. In the negative, solve for \( R_t^* \) under a credit crunch

**Quantitative Analysis and calibration** Bossay, Collard and Smets put their model in a RBC model to relate the Systemic Bank Crises into a business cycle model

- Production function: \( F(k_t, h_t; z_t) \equiv z_t k_t^\alpha h_t^{1-\alpha} \) with \( \alpha \in (0, 1) \)
- Utility function: \( u(c_t, h_t) = \frac{1}{1-\sigma} \left( c_t - \frac{\gamma h_t^{1+\sigma}}{1+\sigma} \right)^{1-\sigma} \)
- Cdf of bank skills: \( \mu(p) = p^\lambda \)
- Real economy: standard calibration on US (annual) post–WII data

67
• Financial sector \((\gamma, \theta, \lambda)\) is calibrated so that:
  
  – Crisis probability is 2.5%
  – Average interest rate spread is 1.71%
  – Average corporate loan rate of 4.35%

  See their paper for more precision.

6.3.4 Example on what went wrong with securitisation

Take a portfolio of two loans to two households which are put together into an asset back security with a "senior tranche" and an "equity tranche"

<table>
<thead>
<tr>
<th></th>
<th>Senior Tranche</th>
<th>Equity Tranche</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both borrowers pay back</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>One defaults</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>The two default</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Let’s now consider the ex ante value of each tranche in three scenarios:

1. Both borrowers have a 10% risk of not paying back for independent reasons (typically in the US, health risks or divorce)

2. Credit risk increases to 20% but still for independent reasons

3. Credit risk is at 20% but is now correlated (due to house prices)

The ex ante value of the tranches are

<table>
<thead>
<tr>
<th></th>
<th>Senior Tranche</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 independent 10% risks</td>
<td>99</td>
<td>81</td>
</tr>
<tr>
<td>2 independent 20% risks</td>
<td>96</td>
<td>64</td>
</tr>
<tr>
<td>2 correlated 20% risks</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

While in the first two cases, changes in credit risk have little effects in the senior tranches (which typically got AAA ratings because they were protected by the equity tranches), in the third case, where the risk is common/macroeconomic, there is no protection of the senior tranches. This was key in the spreading of the "sub-prime" crisis into a systemic crisis.
7 Financial regulation

Outline

1. Introduction
   - Overview of underlying concepts
   - Historical background: Basel 1, 2, 2.5 and 3

2. Why capital/solvency regulation

3. Deposit insurance

4. Liquidity regulation

5. "Chinese walls" regulations

7.1 Introduction

7.1.1 Overview of underlying concepts
   - Corporate governance and the Modigliani-Miller Theorem
   - Leverage and Return on Equity
- Moral hasard, limited liabilities, too big to fail and implicit guaranties
- Systemic risk, fire sale and credit crunch externalities
- Gamble for resurrection
- Bank runs and deposit insurance
- Regulatory arbitrage and shadow banking
- Bail in and bail out
- ...

7.1.2 Historical background on the Basel saga

The set up of Basel 1

1. When Mexico defaults on its debt in 1982, we realise that some banks had lent to Mexico more than their capital. In particular, failure/bankruptcy of Continental Illinois in 1984

2. Need to set minimal capital requirements for banks

3. Need of an international level playing field across countries
   << because capital is more expensive than more senior liabilities (debt, bonds, deposits) there is a risk that "loose" countries would give an advantage to their banks.
   Discuss / Modigliani-Miller theorem

4. Long international bargaining in Basel converges to the Basel 1 regulation: 8 % capital (including 4% or tier one, and 4% of tier 2) exclusively for credit risk. Mexico default in 1982, Agreement in1988, binding in 1993.

The limits of Basel 1

- Other risks than credit risk:
  - Liquidity risk
  - Maturity mismatch
  - Counterparty risk
  - Definition of classes of credit risk, especially / sovereign risk
  - Off balance sheet risks

- Regulatory arbitrage

- Pro-cyclicality
  Kashyap, Rajan and Stein.
**Basel 2** The industry (the banks) have obtained that they could base their capital on their own models of risk. Basel 2 was introduced in 2004 with the objective of becoming binding in 2007, 3 pillars approach:

1. Pillar 1: Risk definition (broader than Basel 1) including, credit risk, operational risks and market risks

2. Pillar 2: the processes through which the monitoring by regulators is processed

3. Pillar 3: / market discipline, i.e. how banks should release the the market their reliance of with the capital requirements.

Major failure of this regulation philosophy completely turned on its head by the sub-prime/Lehman crisis.

**Basel 3** Launched in 2010-2011 even as Basel 2 was not yet binding following the "in the crisis momentum" of the G20 in 2009. Progressive implementation from 2013 to 2018.

New features:

1. Higher "risk weighted" capital requirements:
   - 4.5 % of common equity (2 % in Basel 2)
   - 6% of Tier 1 capital, i.e. common equity and retained earnings (4% in Basel 2)
   - Possibility of additional country level capital buffers if credit boom (2.5 %)

2. Leverage limit in absolute term, minimal 3 % of Tier 1 of total asset in capital.

3. New liquidity requirements
   - LCR: a bank should withstand 30 days cash outflows of liquidity stress
   - Net Stable Funding ratio

4. **Still in debate**, additionnal capital requirements for SIFIs (systemic importance financial institutions), who "benefit" from de facto too big too fail insurance by governements

Gradual implementation until in order to limit the deleveraging/credit crunch externality...

However the risk of "stigma" can accelerate the recap of banks.
7.2 Why capital/solvency regulation

Capital regulation consists of requiring a minimum level of capital on various forms of financial intermediation. Typically, the reserve requirements are 2% and the capital on "risky" loans, was, under Basel I/

- 8% on loans to the private sector,
- none of some asset classes sur as govis (government bonds)
- discuss

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td>Deposits</td>
</tr>
<tr>
<td>Gov. bonds</td>
<td>Bonds/CDs</td>
</tr>
<tr>
<td>Reserves</td>
<td>Capital</td>
</tr>
</tbody>
</table>

7.2.1 Skin in the game/cushion against losses and limited liabilities

This echoes the Holmstrom-Tirole model we have seen in Lecture 5. Because capital is the most junior liability (i.e. the residual value once the asset are sold and the more senior debts are re-imbursed), in the event assets are depreciated, the first one to loose are the owners of capital. Hence need to secure a cushion to absorb losses before bond holders and depositors would be at risk.

Corporate governance: what is a debt contract Definition: repayment is independent of cash flows.

If the cash flows are insufficient,

1. all assets go to lenders
2. lenders get control of the firm

The costly state verification contract is a standard debt contract (and it is optimal, i.e. costly audit is engaged only of the borrowers declares having insufficient cash, see contract theory classes)

Discuss.

A model of limited liabilities 3 Assets, two dates $t = 1, 2$

- Storage Asset : return $\tau$
- Risky Asset : Return $R^*$ at date 2

$$R^* = \begin{cases} 
R \text{ prob. } \pi \\
0 \text{ prob. } 1 - \pi 
\end{cases}$$

Fixed supply $X_R$. Price $P$ in terms of period 1 goods.
• **Safe Asset with decreasing return**: riskless return $r_S$ liabilities of firms with production function $f(X_S) = (X_S^{1-\eta} - 1)/(1 - \eta)$.

3 Agents
Households
• Risk neutral
• Consume at date 2
• Endowment $W^H$
• Information: do not know $f(.)$ nor $X_R$ nor $\pi$. Prices are observable.
• Do not have access to the market for the risky asset. Choice:
  - either store at rate $\tau$
  - or lend to intermediaries an amount $B$ at rate $r$ with a *debt contract*

$$\max_{c,S,B} E \left[ c^H \right]$$

$$S + B \leq W^H \quad \text{(at date 1)}$$
$$c^H \leq \rho B + \tau S \quad \text{(at date 2)}$$

$\rho = r$ with probability $\pi$
$\rho = \text{residual unit value of Intermediaries with probability } 1 - \pi$

Intermediaries
• Risk neutral
• Consume at date 1 and 2, discount rate $\beta < 1/\tau$ (*Impatient*).
• Endowment $W^F$. Utility from intermediation $U$.
• Choice
  - either invest in the risky asset $X_R$
  - or lend to Blue Chips firms an amount $X_S$
• Subject to capital requirement $E \geq \Delta PX^R$
Balance sheet of Financial Intermediaries

\[
\begin{array}{c|c}
\text{Assets} & \text{Liabilities} \\
X_S & E \geq \Delta PX_R \\
PX_R & B \\
\end{array}
\]

\[
\max_{E, B, X_R, X_S} c_1^F + \beta E \left[ c_2^F \right] + U
\]

s.t. \( PX_R + X_S = B + E \)
\( c_1^F \leq W^F - E \)
\( c_2^F \leq \max\{R^e X_R + r_S X_S - r B, 0\} \)
\( E \geq \Delta PX_R \)

Initial sellers (not important)
Sell the quantity of assets \( X^R \) and consume only in period 1.

\( c^i = PX_R \)

The market segmentation

Resolution
Return of intermediaries at period 2

\[
\max\{R X_R + r_S X_S - r B, 0\}
\]

Intermediaries always default in the bad state of the world, when the aggregate return is \( R^e = 0 \). \( r = r_S \).
The main pricing equation of the model

\[ P = \frac{\beta \pi R}{\Delta + \beta \pi r (1 - \Delta)} \]

\[ P = P \left( r, \pi, \frac{\Delta}{\pi} \right) \leq P^0 \text{ for } \Delta > 0 \]

\[ \Delta = 0 \Rightarrow P^0 = \frac{R}{r} \text{ (Allen and Gale)} \]

**No-arbitrage condition for households**

\[ \pi r + (1 - \pi) \frac{r_s X_S}{B} = \tau \]

Solution when banks’ leverage is known

**Proposition:** Both the level \( B \) and the risk premium increase when capital requirement decrease (i.e. \( \Delta \) decreases)

\[ \frac{\partial B}{\partial \Delta} < 0 \text{ and } \frac{\partial(r - \tau)}{\partial \Delta} < 0 \]

\( \Delta \downarrow \Rightarrow \) increase in leverage (Adrian and Shin among others) but higher risk premium.

Intuition: \( \Delta \downarrow \): more incentives to take risks\( \Rightarrow P \uparrow \) and portfolio of intermediaries twisted toward risky assets.

\( \Rightarrow \) lower liquidation value \( \Rightarrow \) higher risk premium required.

### 7.2.2 Current debate on capital regulation

**Mostly about pro-cyclicality and the cost of capital**

- The industry claims that raising capital implies reducing credit supply and therefore growth, and this is happening at the worst possible time while you have debt overhang and anemic growth
  
  This is because "capital" would cost more than other forms of funding

- Some economists (Martin Hellwig, Stiglitz,...) argue that capital is not more expensive

- Others (Rochet, Franklin Allen,...) debt is less expensive for bad reasons such as tax deductibility on debt and this should be corrected

- Kashyap, Rajan and Stein stress capital requirements should be procyclical (i.e. 10% in booms and 8% in recession) to limit the fire sale /credit crunch externality. An alternative is to force banks to buy capital insurance from "deep pockets".
7.3 Deposit insurance

Essentially to prevent bank runs
   Diamond-Dybvig model of bank runs
   Depositors are nearly always and everywhere explicitly or implicitly insured

7.4 Liquidity regulation

See the Maturity rat race section of Lecture 6

7.5 "Chinese walls" regulations

Meant to address contagion.
   Extreme case of the US response great depression

1. Interstate branching act

2. Glass-Steagall act: separation of deposit banks and investment banks;
   Introduced in 1932 and formally abolished in 1999 under Clinton administration (with Rubin, former Goldman Sachs; at the treasury)

Responses to the current crisis
Volker plan in the US
Vickers report in the UK
New bank law in France, in parliament in February 2013
"The French view: our universal banks (i.e. that do all) have proven their strength"
8 The crisis of the euro area/euromonics

Outline

1. Optimal currency areas
2. What went wrong: the build up of macroeconomic imbalances
3. The crisis and its resolution

8.1 Optimal currency areas

Main source: "Economics of Monetary Union" by Paul de Grauwe, Oxford

8.1.1 The costs of giving up monetary policy

Consider two countries in a monetary union. Single money and single interest rate.

Adjustments to asymmetric shocks  Take two countries subject to an asymmetric demand shock. What adjustment mechanism can arise

- wage flexibility
- labour mobility
- change in wages/prices through the exchange rate

Hard and soft defaults  In the event of large debt (either public or private) you may have the choice between "default" and issuing money to buy debt

Asymmetric shock and debt dynamics  The asymmetric demand shock can be amplified by its effects on fiscal balances and public debt.

Insurance mechanisms 1: a single budget  Permanent and temporary shocks?

Insurance mechanism 2: financial integration  Permanent and temporary shocks?

8.1.2 Reconsidering the costs of giving up monetary policy

Are asymmetric shock relevant/material/frequent?

- There is a world (and a OECD (and a European)) business cycle, [see Agresti and Mojon]
- Trade integration >> more symmetry: The European Commission view
• Trade integration >> less specialisation; the "Krugman view"
• Which one dominates?

Can devaluations adjust asymmetric shocks? "beggar thy neighbor policies"

• Initial stimulus
• What is inflation through higher import prices?
• Uncertainty of the effects (size, timing, persistence) and response to temporary asymmetric shocks
• Time inconsistency (of stimulus monetary policies) in open economies

At the end of the day, exchange rate may turn out to be "destabilizing" in as of itself

Costs of monetary unions and openness of the economy

• Likelihood of asymmetric shocks and monetary union participation (i.e. +/- trade integration)
• Effectiveness of an exchange rate policy and openness
  More open economies are more sensitive to exchange rate fluctuations (more effective on demand) yet more effects on prices as well so more costly in the sense of more price variability

8.2 What went wrong in the euro area?
Chen, Milesi-Ferretti and Tressel Economic Policy January 2013

8.2.1 Competitiveness and trade imbalances
8.2.2 Capital flows
8.2.3 One size fit all monetary policy?

8.3 The crisis and its resolution
8.3.1 Sovereign debt crises since 2010
Greece,...
  Spill over to the private sector (financial fragmentation)

8.3.2 The crisis resolution institutions on the fiscal side
EFSF
  ESM (discuss its size)
8.3.3 Monetary policy

- Target II balances: compare the consolidated and the unconsolidated balance sheet of the Eurosystem

- Financial fragmentation (See Gilchrist and Mojon)

The unconditional monetary policy response  FRFA
   VLTRO
   SMP
   OMT
9 Preparing for the exam

9.1 The main models/concepts of the class

- AD-AS and the roles of monetary and fiscal policies. Its representation in the Benigno model
- Price stickiness and the real effects of monetary policy
- Time inconsistency, inflation biases
- The Taylor rule
- The dynamic of public debt
- Multiple equilibria and public default
- The Zero Lower Bound
- Non conventional monetary policies (Forward guidance, QE, VLTRO, OMT)
- Effects of LSAP
- VLTRO and the interbank market
- Financial intermediation
- Asset price / debt cycles
- Maturity mismatch
- Limited liabilities
- Optimal currency areas
- Real exchange rate, trade imbalances and capital flows

9.2 Prep exam

9.2.1 Exercise 1: Money and the public debt

Public debt dynamic

\[ B_t = (1 + R_{t-1})B_{t-1} + G_t - T_t - (M_t - M_{t-1}) \]

**Question 1:** divide debt by nominal GDP and show how public debt depends on inflation and the real growth of the economy.

**Answer:**

\[ \frac{B_t}{P_t Y_t} = (1 + R_{t-1}) \frac{P_{t-1} Y_{t-1}}{P_t Y_t} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} + \frac{D_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t} \]
\[ b_t = \frac{1 + R_{t-1}}{1 + \pi_t + g_t} b_{t-1} + d_t - \Delta m_t \]

**Question 2:** Why does an increase in base money help the budget constraint of the government?

**Question 3:** What is the limit of this type of financing of public debt? Can the quantitative theory of money, which relates money to economic activity and prices as

\[ M_t v_t = P_t Y_t \]

where \( v_t \) is the velocity of money circulation, help you illustrate these limits?

**Question 4:** Assume the following balance sheets of the central bank and of the banking system, where reserve requirements amount to 10% of deposits:

**Banking system**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves 10</td>
<td>Deposits 100</td>
</tr>
<tr>
<td>Loans 90</td>
<td>CDs 20</td>
</tr>
<tr>
<td>Bonds 40</td>
<td>Capital 20</td>
</tr>
</tbody>
</table>

**Central bank**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves 10</td>
<td>Reserves 10</td>
</tr>
<tr>
<td>Gold 6</td>
<td>Bills 10</td>
</tr>
<tr>
<td>Forex Res. 8</td>
<td>Capital 10</td>
</tr>
<tr>
<td>Bonds 6</td>
<td></td>
</tr>
</tbody>
</table>

In this system, define two concepts of money. Which one of the two is relevant for the budget constraint of the government? Which one is relevant for the quantity theory of money?

**Question 5:** Comment the evolution of central banks balance sheets, M3 and inflation over the last 10 years and relate it to your answer to question 4.

INSERT M3, inflation and the CB balance sheet.

**9.2.2 Exercise 2: The zero lower bound**

Assume the economy exists for two periods and its prices, output and natural level of output are given by the following 3 equations where letters with caps refer to period 2 realisations: e.g. \( \bar{p} \) is the price level in period 2.

\[ p - p^\circ = \kappa (y - y_n) \]

\[ y = \bar{y} + (g - \bar{g}) - \sigma [\bar{v} - (\bar{p} - p)] + \sigma \ln \beta \]

\[ y_n \approx \frac{1 + \eta}{\eta + 1/\sigma} a + \frac{1/\sigma}{\eta + 1/\sigma} g - \frac{1}{\eta + 1/\sigma} \mu \]

**Question 1:** Describe in the \((y, p)\) the equilibrium and then the effects of a positive shock to current productivity \(a\), a negative shock to public spending \(g\).
Question 2: Assume a negative demand shock has shifted the demand curve to the left. What are the effects on output and prices? How can monetary and fiscal policy be activated to restore the equilibrium?

Question 3: Assume a negative supply shock has shifted the supply curve to the left. What are the effects on output and prices? What can be achieved with a change in the nominal interest rate? Confronted to this situation, would you expect the Fed and the ECB to have the same response? Why?

Question 4: Assume that a negative demand shock has moved the demand curve to the left while the nominal interest rate is equal to zero. What can be done to influence demand? What can be done to influence supply?

Question 5: Is this model helpful to understand the non-conventional policies of the Federal Reserve, the Bank of Japan, the Bank of England and the European Central Bank?

9.2.3 Exercise 3: Financial intermediation

Assume:
3 Assets, two dates t = 1, 2

- **Storage Asset**: return \( \tau \)
- **Risky Asset**: Return \( R^* \) at date 2
  \[
  R^* = \begin{cases} 
  R \text{ prob. } \pi \\
  0 \text{ prob. } 1 - \pi 
  \end{cases}
  \]

  Fixed supply \( X_R \). Price \( P \) in terms of period 1 goods.

- **Safe Asset with decreasing return**: riskless return \( r_S \) liabilities of firms with production function \( f (X_S) = (X_S^{1-\eta} - 1) / (1 - \eta) \).

Households

- Risk neutral
- Consume at date 2
- Endowment \( W^H \)

- Information: do not know \( f (\cdot) \) nor \( X_R \) nor \( \pi \). Prices are observable.
- Do not have access to the market for the risky asset. Choice:
  - either store at rate \( \tau \)
  - or lend to intermediaries an amount \( B \) at rate \( r \) with a debt contract

\[
\begin{align*}
\rho &= r \text{ with probability } \pi \\
\rho &= \text{ residual unit value of Intermediaries with probability } 1 - \pi
\end{align*}
\]

Banks
• Risk neutral
• Consume at date 2, discount rate $\beta < 1/\tau$ (Impatient).
• Utility from intermediation $U$.
• Borrow from households and choose
  - either invest in the risky asset $X_R$
  - or lend to Blue Chips firms an amount $X_S$

**Question 1:** write the balance sheet and the profit function of the bank

**Answer:**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_S$</td>
<td>$PX_R$</td>
</tr>
<tr>
<td>$PX_R$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

$$
\max_{B,X_R,X_S} \beta \max \{R^* X_R + r S X_S - \tau B, 0\} + U
\text{ s.t. } PX_R + X_S = B
$$

**Question 2:** What the arbitrage condition for households?

**Answer:**

$$
\pi r + (1 - \pi) \frac{r S X_S}{B} = \tau
$$

**Question 3:** How can household deduct $\pi$ from $P$ the price level of the risky asset?

$$
P = \frac{\pi R}{\pi r}
$$

**Question 4:** How does limited liabilities affect the price of the risky asset?

$$
P^{SP} = \frac{\pi R}{\tau}
$$

**Question 5:** How are households covered from the risks taken by the financial intermediaries? What would be the effects of having risk adverse households?

**Question 6:** What would be the effects of deposit insurance? What would be the effects of capital requirements?
9.2.4 General interest questions

- Central banks typically hold gold. And gold prices typically increase during crisis. How can this matter in the event of a financial crisis?

- A recent estimation shows that the euro dollar exchange rate has tow main determinants as follows:

\[ EURDOL_t = 0.09(i_{t+24}^{EZ} - i_{t+24}^{US}) - 0.03(i_t^{ES} - i_t^{DE}) \]

Explain each of the terms.

Given that \( i_{t+24}^{EZ} = 0.4, i_{t+24}^{US} = 0.1; i_t^{ES} - i_t^{DE} = 2.2\% \), by how much can the euro appreciate or depreciate in various scenarii.

- Cost and benefits of price level targeting?

- Why would an increase in capital requirements for banks could increase the cost of bank funding? Why could it decrease it?